CS 173, Spring 2016
Examlet 9, Part B

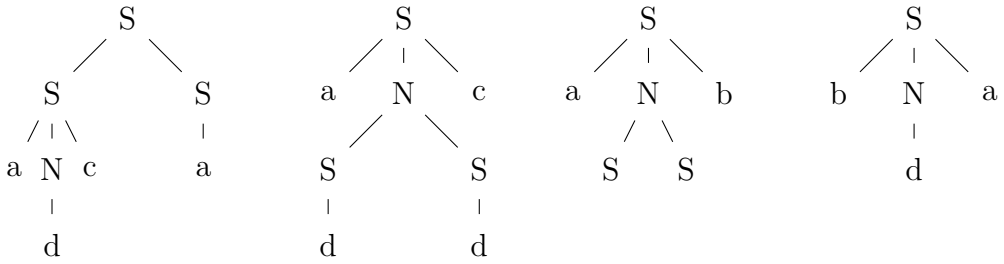
NETID:

FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$\begin{aligned} S &\rightarrow b\ N\ a \mid a\ N\ c \mid a \\ N &\rightarrow S\ S \mid d \end{aligned}$$



2. (4 points) Check the (single) box that best characterizes each item.

The diameter of a full, complete tree of height h.

$\leq h$ ☐ h ☐ $h+1$ ☐

$2h$ ☐ $\leq 2h$ ☐

The level of the root node in a tree of height h.

0 ☐ 1 ☐ $h-1$ ☐ h ☐ $h+1$ ☐

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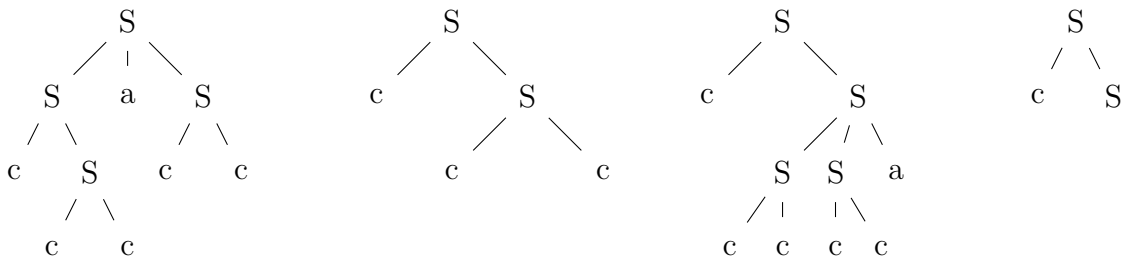
1. (8 points) Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \rightarrow S\ S\ a \mid c\ S \mid c\ c$$



2. (4 points) Check the (single) box that best characterizes each item.

The level of a leaf node in a tree of height h.

0 ☐ 1 ☐ $h-1$ ☐ $\leq h$ ☐ h ☐

$\sum_{k=0}^{n-1} 2^k$ $2^n - 2$ ☐ $2^n - 1$ ☐ $2^{n-1} - 1$ ☐ $2^{n+1} - 1$ ☐

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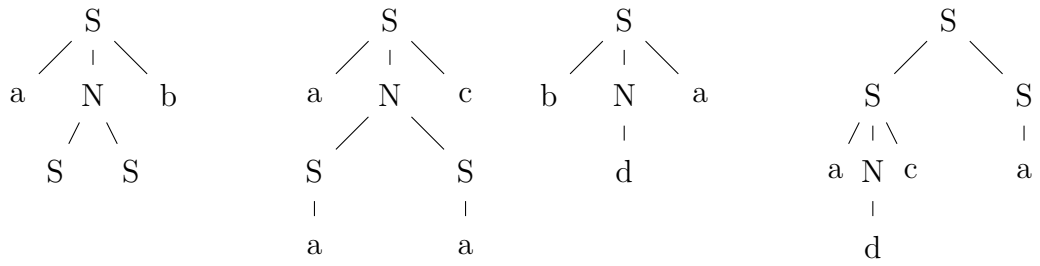
1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$\begin{aligned} S &\rightarrow a\ N\ b \mid a\ N\ c \mid a \\ N &\rightarrow S\ S \mid d \end{aligned}$$



2. (4 points) Check the (single) box that best characterizes each item.

Total number of leaves in a 3-ary tree of height h

3^h	☐	$\leq 3^h$	☐
$\frac{1}{2}(3^{h+1}-1)$	☐	$3^{h+1}-1$	☐

The number of nodes in a binary tree of height h

$\geq 2^h$	☐	$2^{h+1}-1$	☐
$\leq 2^{h+1}-1$	☐	$\geq 2^{h+1}-1$	☐

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1. (8 points) Consider the following grammar G

   $S \to b\ S\ a \mid a\ S\ b \mid c$

   S is the only start symbol. The terminal symbols are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   $b\ b\ a\ c\ b\ a\ b$

   $b\ a\ b\ c\ a\ b\ a$

2. (4 points) Check the (single) box that best characterizes each item.

   The chromatic number of a full 3-ary tree

1	☐	2	☐	≤ 2	☐
3	☐	≤ 3	☐	can't tell	☐

   A tree with n nodes has

n edges	☐	$n-1$ edges	☐	$\leq n$ edges	☐
$n/2$ edges	☐	$\log n$ edges	☐		

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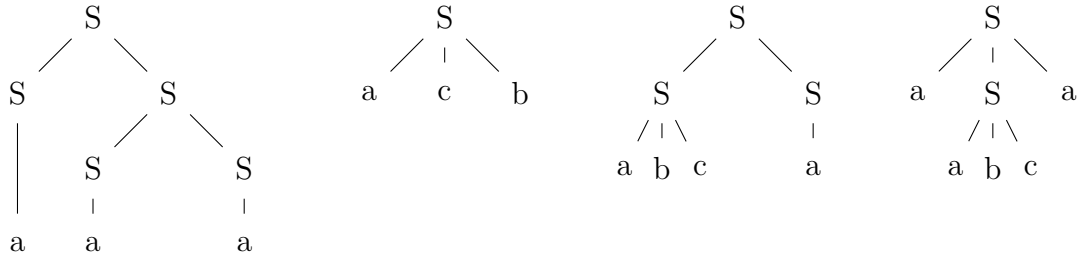
1. (8 points) Here is a grammar with start symbol S and terminals symbols a,b, and c. Circle the trees that match the grammar.

$$S \rightarrow S\ S \mid a\ b\ c \mid a$$



2. (4 points) Check the (single) box that best characterizes each item.

The diameter of a tree of height h.

$\leq h$ ☐ h ☐ $h+1$ ☐

$2h$ ☐ $\leq 2h$ ☐

Total number of leaves in a full and complete 5-ary tree of height h

5^h ☐ $\leq 5^h$ ☐

$\geq 5^h$ ☐ $5^{h+1}-1$ ☐

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1. (8 points) Consider the following grammar G

   $S \to a\ S\ b \mid b\ S\ b \mid c$

   S is the only start symbol. The terminal symbols are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   $b\ a\ b\ c\ b\ b\ b$ $\qquad\qquad$ $a\ b\ c\ b\ a$

2. (4 points) Check the (single) box that best characterizes each item.

   New question which proved a bit too hard. So we accepted a couple other answers for full credit.

   The number of paths in a tree with n nodes

   n ☐ $\qquad$ $2n$ ☐ $\qquad$ $\frac{n(n-1)}{2}$ ☐

   $n(n-1)$ ☐ $\qquad$ n^2 ☐

   A binary tree of height h has at least $2^h - 1$ vertices (nodes).

   true ☐ $\qquad$ false ☐