CS 173, Spring 2016
Examlet 10, Part A

NETID:

FIRST: **LAST:**

Discussion: **Monday 9 10 11 12 1 2 3 4 5**

(15 points) Use (strong) induction to prove the following claim:

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \leq 2\sqrt{n}$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: notice that $(\sqrt{x} - \sqrt{x+1}\ )^2 \geq 0$. What does this imply about $2\sqrt{x}\sqrt{x+1}$?

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(15 points) Let function $f : \mathbb{Z}^+ \to \mathbb{R}$ be defined by

$f(1) = f(2) = 1$

$f(n) = \frac{1}{2} f(n-1) + \frac{1}{f(n-2)}$

Use (strong) induction to prove that $1 \leq f(n) \leq 2$ for all positive integers n.

Hint: prove both inequalities together using one induction.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: $\sum_{p=2}^{n} \frac{1}{p^2} \le \frac{3}{4} - \frac{1}{n}$ for all integers $n \ge 2$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Let function $f : \mathbb{Z}^+ \to \mathbb{R}$ be defined by

$f(1) = 2$

$f(n) = \frac{1}{2}f(n-1) + \frac{5}{2f(n-1)}$

Use (strong) induction to prove that $2 \leq f(n) \leq \frac{5}{2}$ for any positive integer n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Suppose that $0 < q < \frac{1}{2}$. Use (strong) induction to prove the following claim:

Claim: $(1+q)^n \leq 1 + 2^n q$, for all positive integers n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Notice that $1 \leq 2^{n-1}$ for any positive integer n.

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(15 points) Use (strong) induction to prove the following claim. You may use the fact that $\sqrt{2} \leq 1.5$.

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \geq 2\sqrt{n+1} - 2$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: notice that $(\sqrt{x+1} - \sqrt{x+2}\ )^2 \geq 0$. What does this imply about $2\sqrt{x+1}\sqrt{x+2}$?