CS 173, Spring 2016
Examlet 10, Part B

NETID:

FIRST: | **LAST:**

Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x))$?

   Solution: This is false.

   Suppose that $f(x) = g(x) = h(x) = x$. Then $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$, but $f(x)g(x) = x^2$ is not $O(h(x))$.

2. (8 points) Check the (single) box that best characterizes each item.

   Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $f(n)$ be $O(g(n))$?
   no ☐ perhaps ☐ yes ☑

   $T(1) = d$
   $T(n) = 2T(n/2) + c$
   $\Theta(\log n)$ ☐ $\Theta(n)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

   $n!$
   $O(2^n)$ ☐ $\Theta(2^n)$ ☐ neither of these ☑

   $n^{1.5}$ is
   $\Theta(n^{1.414})$ ☐ $O(n^{1.414})$ ☐ neither of these ☑

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Discussion: **Monday 9 10 11 12 1 2 3 4 5**

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(4) = 7 \qquad T(n) = 4T\left(\frac{n}{2}\right) + d$

   (a) The height: $\log_2(n) - 2$

   (b) Number of nodes at level k: 4^k

   (c) Sum of the work in all the leaves (please simplify):
   Each leaf contains the value 7, and there are $4^{\log_2 n - 2} = \frac{1}{16}4^{\log_2 n} = \frac{1}{16}n^2$ leaves. So the sum is $\frac{7}{16}n^2$.

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $30\log(n^{17}) \qquad \sqrt{n} + n! + 18 \qquad \frac{n \log n}{7} \qquad (10^{10^{10}})n \qquad 0.001n^3 \qquad 2^n \qquad 8n^2$

   Solution:

   $30\log(n^{17}) \ll (10^{10^{10}})n \ll \frac{n \log n}{7} \ll 8n^2 \ll 0.001n^3 \ll 2^n \ll \sqrt{n} + n! + 18$

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1. (7 points) Suppose that f and g are functions from the reals to the reals, such that f is $\Theta(g)$. Must f be $O(g)$?

   Solution: This is true. The definition of Θ is that the big-O relationship holds in both directions.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + c$

$\Theta(\log n)$ [√] $\Theta(n)$ [ ]
$\Theta(n \log n)$ [ ] $\Theta(n^2)$ [ ]

$T(1) = d$
$T(n) = T(n-1) + n$

$\Theta(n)$ [ ] $\Theta(n^2)$ [√] $\Theta(n \log n)$ [ ] $\Theta(2^n)$ [ ]

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ [√] $k = m$ [ ]
$k > m$ [ ] $km = 1$ [ ]

3^n is

$\Theta(5^n)$ [ ] $O(5^n)$ [√] neither of these [ ]

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 7.

   $T(1) = 5 \qquad T(n) = 3T\left(\frac{n}{7}\right) + n^2$

   (a) The height: $\log_7 n$.

   (b) The number of leaves (please simplify): $3^{\log_7 n} = 3^{\log_3 n \log_7 3} = n^{\log_7 3}$

   (c) Value in each node at level k: $(\frac{n}{7^k})^2$

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $2^n + 3^n \qquad n^3 \qquad 100 \log n \qquad 3^{31} \qquad 3n \log(n^3) \qquad 7n! + 2 \qquad 173n - 173$

   Solution:

   $3^{31} \ll 100 \log n \ll 173n - 173 \ll 3n \log(n^3) \ll n^3 \ll 2^n + 3^n \ll 7n! + 2$

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 4.

   $T(4) = 7 \qquad T(n) = 2T\left(\frac{n}{4}\right) + d$

   (a) The height: $\log_4(n) - 1$

   (b) Number of nodes at level k: 2^k

   (c) Sum of the work in all the leaves (please simplify):
   Each leaf contains the value d, and there are $2^{\log_4 n - 1} = \frac{1}{2}2^{\log 4} = \frac{1}{2}\sqrt{n}$ leaves. So the sum is $\frac{d}{2}\sqrt{n}$.

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $n \qquad n\log(17n) \qquad \sqrt{n}+2^n+18 \qquad 8n^2 \qquad 2^n+n! \qquad 2^{\log_4 n} \qquad 0.001n^3+3^n$

   Solution:

   $2^{\log_4 n} \ll n \ll n\log(17n) \ll 8n^2 \ll \sqrt{n}+2^n+18 \ll 0.001n^3+3^n \ll 2^n+n!$

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x)h(x))$?

   Solution: This is true. Since $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$, there are positive reals c, k, C and K such that $0 \leq f(x) \leq ch(x)$ and $0 \leq g(y) \leq Ch(y)$ for every $x \geq k$ and $y \geq K$.

   But then if we let $p = cC$, we have $O \leq f(x) \leq ph(x)h(x)$ for every $x \geq \max(k, K)$.

2. (8 points) Check the (single) box that best characterizes each item.

   Suppose $f(n)$ is $\Theta(g(n))$.
   Will $g(n)$ be $\Theta(f(n))$? no ☐ perhaps ☐ yes ☑ ($\surd$)

   $T(1) = c$
   $T(n) = 3T(n/3) + n$ $\Theta(n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(n \log n)$ ☑ ($\surd$) $\Theta(2^n)$ ☐

   Suppose $f(n)$ is $O(g(n))$.
   Will $g(n)$ be $O(f(n))$? no ☐ perhaps ☑ ($\surd$) yes ☐

   $n^{log_2 5}$ grows faster than n^2 ☑ ($\surd$) slower than n^2 ☐ at the same rate as n^2 ☐