CS 173, Spring 2016
Examlet 10, Part B

NETID:

FIRST: | **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $f(n)$ be $O(g(n))$? — no ☐ perhaps ☐ yes ☐

$T(1) = d$
$T(n) = 2T(n/2) + c$ — $\Theta(\log n)$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

$n!$ — $O(2^n)$ ☐ $\Theta(2^n)$ ☐ neither of these ☐

$n^{1.5}$ is — $\Theta(n^{1.414})$ ☐ $O(n^{1.414})$ ☐ neither of these ☐

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(4) = 7 \qquad T(n) = 4T\left(\frac{n}{2}\right) + d$

   (a) The height:

   (b) Number of nodes at level k:

   (c) Sum of the work in all the leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $30\log(n^{17})$ $\quad$ $\sqrt{n}+n!+18$ $\quad$ $\frac{n\log n}{7}$ $\quad$ $(10^{10^{10}})n$ $\quad$ $0.001n^3$ $\quad$ 2^n $\quad$ $8n^2$

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Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

1. (7 points) Suppose that f and g are functions from the reals to the reals, such that f is $\Theta(g)$. Must f be $O(g)$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = T(n/2) + c$

$\Theta(\log n)$ ☐ $\Theta(n)$ ☐
$\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

$T(1) = d$
$T(n) = T(n-1) + n$

$\Theta(n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(2^n)$ ☐

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐
$k > m$ ☐ $km = 1$ ☐

3^n is

$\Theta(5^n)$ ☐ $O(5^n)$ ☐ neither of these ☐

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 7.

   $T(1) = 5 \qquad T(n) = 3T\left(\frac{n}{7}\right) + n^2$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $2^n + 3^n \qquad n^3 \qquad 100\log n \qquad 3^{31} \qquad 3n\log(n^3) \qquad 7n! + 2 \qquad 173n - 173$

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 4.

   $T(4) = 7$ $\qquad$ $T(n) = 2T\left(\frac{n}{4}\right) + d$

   (a) The height:

   (b) Number of nodes at level k:

   (c) Sum of the work in all the leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   n $\qquad$ $n\log(17n)$ $\qquad$ $\sqrt{n}+2^n+18$ $\qquad$ $8n^2$ $\qquad$ $2^n+n!$ $\qquad$ $2^{\log_4 n}$ $\qquad$ $0.001n^3+3^n$

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x)$ is $O(h(x))$. Must $f(x)g(x)$ be $O(h(x)h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

Suppose $f(n)$ is $\Theta(g(n))$.
Will $g(n)$ be $\Theta(f(n))$? no ☐ perhaps ☐ yes ☐

$T(1) = c$
$T(n) = 3T(n/3) + n$ $\Theta(n)$ ☐ $\Theta(n^2)$ ☐ $\Theta(n \log n)$ ☐ $\Theta(2^n)$ ☐

Suppose $f(n)$ is $O(g(n))$.
Will $g(n)$ be $O(f(n))$? no ☐ perhaps ☐ yes ☐

$n^{log_2 5}$ grows faster than n^2 ☐ slower than n^2 ☐ at the same rate as n^2 ☐