CS 173, Spring 2016
Examlet 12, Part A

NETID:

FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

(a) (9 points) Suppose that G is a graph with 30 nodes. Use proof by contradiction to show that two of the nodes have the same degree.

Solution: Suppose G is a graph with 30 nodes, all of which have different degrees. Since there are 30 nodes, the degree of each node cannot be larger than 29. Since there are only 30 numbers between 0 and 29, and we've assumed that all nodes have different degrees, this means that one node (call it p) has degree 0 and another node (call it q) has degree 29. Since q has degree 29, it must be connected to all of the other nodes. But q can't be connected to p because p has degree 0. So we have a contradiction.

(b) (6 points) Suppose a car dealer is planning to buy a set of Civics, Accords, and Fits (three kinds of cars). The dealer will buy ten cars in total and can buy any number of each type. How many different choices does he have? The sets are unordered, so three Civics and seven Fits is the same as seven Fits and three Civics.

Solution: Using the formula for combinations with repetition, there are

$$\binom{10+2}{2}$$

choices.

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(a) (9 points) Ignatius Eggbert flips a coin 12 times. The coin is fair, i.e. equal chance of getting a head vs. a tail. What is the chance that he gets 11 or more heads? Give an exact formula; don't try to figure out the decimal equivalent. Briefly explain your answer and/or show work.

Solution: There are 2^{12} sequences of 10 coin flips. There are $\binom{12}{11}$ ways to pick 11 of these flips to contain a head. And there is exactly one sequence that contains 12 heads. So the chance of getting 11 or more heads is

$$\frac{1 + \binom{12}{11}}{2^{12}}$$

(b) (6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every cat c, if c is not fierce or c wears a collar, then c is a pet.

Solution: There exists a cat c that is either not fierce or wears a collar and is not a pet.

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(a) (9 points) Use proof by contradiction to show that $\sqrt{6} - \sqrt{2} > 1$

Solution: Suppose not. That is, suppose that $\sqrt{6} - \sqrt{2} \leq 1$.

Then $\sqrt{6} \leq 1 + \sqrt{2}$.

Since both sides are positive, we can square both sides to get $6 \leq 1 + 2\sqrt{2} + 2$. So $3 \leq 2\sqrt{2}$. Squaring both sides again, we get $9 \leq 8$, which is clearly false. Since we have a contradiction, our assumption at the start must have been wrong. So $\sqrt{6} - \sqrt{2} > 1$.

(b) (6 points) Suppose that A is a set containing p elements and B is a set containing n elements. How many functions are there from A to $\mathbb{P}(B)$? How many of these functions are one-to-one?

Solution: $\mathbb{P}(B)$ contains 2^n elements. So the total number of functions from A to $\mathbb{P}(B)$ is $(2^n)^p$. The number of one-to-one functions is $\frac{2^n!}{(2^n-p)!}$.

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(a) (9 points) CMU's new robotic hummingbird Merrill travels in 3D. Each command makes it move one foot in a specified cardinal direction e.g. up/down, north/south, or east/west, but not diagonally. How many different sequences of 30 commands will get Merrill from position $(1, 10, 3)$ to position $(10, 6, 20)$? Briefly explain your answer and/or show work.

Solution: We'll need 9 commands that increase the first coordinate, 4 that decrease the second coordinate, and 17 that increase the third coordinate. There are $\binom{30}{9}$ ways to pick which of the 30 commands changes the first coordinate, and then $\binom{21}{4}$ choices for when to change the second coordinate. So our total choices are

$$\binom{30}{9}\binom{21}{4}$$

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(b) (6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a mushroom f such that f is not poisonous or f is blue.

Solution: For every mushroom f, f is poisonous and f is not blue.

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(a) (9 points) Ignatius Eggbert flips a coin 10 times. The coin is fair, i.e. equal chance of getting a head vs. a tail. What is the chance that he gets exactly 7 heads? Give an exact formula; don't try to figure out the decimal equivalent. Briefly explain your answer and/or show work.

Solution: There are 2^{10} sequences of 10 coin flips. There are $\binom{10}{7}$ ways to pick 7 of these flips to contain a head. So the chance of getting 7 heads is

$$\frac{\binom{10}{7}}{2^{10}}$$

(b) (6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a jedi j such that j wields a red light saber and j is not fighting for the Dark Side.

Solution: For every jedi j, j does not wield a red light saber or j is fighting for the Dark Side.

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(a) (9 points) Donald Knuth has proposed a replacement for conventional resistor codes. In the new system, each resistor has 10 stripes. Each stripe can be either red, blue, or green. The type of resistor is determined by the total amount of each color. E.g. two resistors with 4 red, 5 blue, and 1 green are the same, regardless of the order in which those stripes appear. How many different types of resistor can this code represent?

Solution: This is a combination with repetition problem. We have three types of object (i.e. two dividers) and 10 objects whose type we must choose. So we are choosing 2 positions for the dividers among 12 total positions. So our total number of choices is

$$\binom{10+2}{2} = \binom{12}{2} = \binom{12}{10}$$

(b) (6 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

> There is a bug b, such that for every plant p, if b pollinates p and p is showy, then p is poisonous.

Solution: For every bug b, there is a plant p, such that b pollinates p and p is showy, but p is not poisonous.