CS 173, Spring 2016
Examlet 12, Part B

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FIRST: **LAST:**

Discussion: Monday 9 10 11 12 1 2 3 4 5

(9 points) Let $f : X \to Y$ be any function, and let A and B be subsets of X. For any subset S of X define its image $f(S)$ by $f(S) = \{f(s) \in Y \mid s \in S\}$. Is it the case that $f(A) \cap f(B) = f(A \cap B)$? Informally explain why this is true or give a concrete counter-example showing why it is not.

Solution: Let $X = \{a, b\}$ and $Y = \{c\}$. Define $f : X \to Y$ by $f(x) = c$ for all $x \in X$. Suppose that $A = \{a\}$ and $B = \{b\}$. Then $A \cap B = \emptyset$, $sof(A \cap B) = \emptyset$. But $f(A) \cap f(B) = \{c\}$

(6 points) Check the (single) box that best characterizes each item.

How many ways can I choose 5 bagels from among 10 varieties, if I can have any number of bagels from any type?

- $\frac{10!}{5!5!}$ ☐
- $\frac{14!}{10!4!}$ ☐
- $\frac{14!}{9!5!}$ ☑ √
- $\frac{15!}{10!5!}$ ☐
- 10^5 ☐
- 5^{10} ☐

Pascal's identity states that $\binom{n+1}{k}$ is equal to

- $\binom{n}{k} + \binom{n}{k+1}$ ☐
- $\binom{n}{k} + \binom{n-1}{k}$ ☐
- $\binom{n}{k} + \binom{n}{k-1}$ ☑ √

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is

- an integer ☐
- a set of integers ☑ √
- one or more integers ☐
- a power set ☐

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Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid (p - x)^2 + (q - y)^2 = 4\}$
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

Describe (at a high level) the elements of $f(3, 5)$:

Solution: The circle with radius 2 centered at $(3, 5)$.

$f(0, 0) \cap f(0, 4) =$

Solution: $\{(0, 2)\}$

The cardinality of (aka the number of elements in) T is:

Solution: infinite

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: No, this is not a partition of the plane. The output of f is never the empty set. And these circles jointly cover the whole plane. However, distinct circles share points, so T contains partly overlapping sets.

(2 points) Check the (single) box that best characterizes each item.

$|\{A \subseteq \mathbb{Z}_4 \ : \ |A| \text{ is even}\}|$ 1 ☐ 6 ☐ 7 ☐ 8 ☑ infinite ☐

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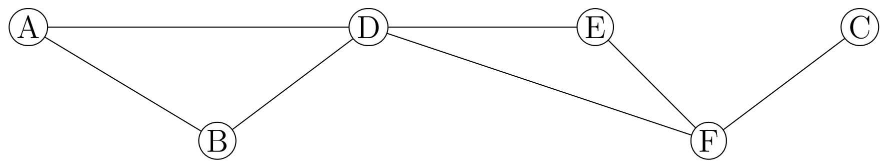
Graph G is at right.
V is the set of nodes in G.



Define $f : V \to \mathbb{P}(V)$ by $f(p) = \{n \in V \ : \ \deg(n) \leq \deg(p)\}$, where deg(n) is the degree of node n. Let $P = \{f(p) \mid p \in V\}$.

(6 points) Fill in the following values:

f(A) =

Solution: $\{A, B, C, E\}$

f(C) =

Solution: $\{C\}$

P =

Solution: $\{\{C\}, \{A, B, C, E\}, \{A, B, C, E, F\}, \}, \{A, B, C, D, E, F\}\}$

(7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

Solution: No, it is not a partition. It doesn't contain the empty set (good), and it covers all of V (good), but there is partial overlap among its members (bad).

(2 points) Check the (single) box that best characterizes each item.

For all sets A and B, $\mathbb{P}(A \cap B) \subseteq \mathbb{P}(A \cup B)$. always [√] sometimes [ ] never [ ]

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(9 points) Define $f : \mathbb{Z} \times \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z})$ by $f(x, k) = \{y \in \mathbb{Z} : x = y + kn \text{ for some } n \in \mathbb{Z}\}$. Suppose that $k|p$. Compare $f(r, k)$ and $f(r, p)$. Justify your answer.

Solution: $f(r, p)$ is a subset of $f(r, k)$. Because k divides p, two numbers that differ by a multiple of p must also differ by a multiple of k, but not vice versa. So each equivalence class mod k is the union of several equivalence classes mod p.

(6 points) Check the (single) box that best characterizes each item.

A partition of a set A contains $\emptyset$

always ☐ sometimes ☐ never ☑ $\surd$

How many ways can I choose 6 bagels from among 8 varieties, if I can have any number of bagels from any type?

$\frac{8!}{6!2!}$ ☐ $\frac{13!}{6!7!}$ ☑ $\surd$ $\frac{14!}{9!5!}$ ☐

$\frac{14!}{6!7!}$ ☐ 8^6 ☐ 6^8 ☐

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(3)$ is

a rational ☐ a power set of rationals ☐

a set of rationals ☑ $\surd$ undefined ☐

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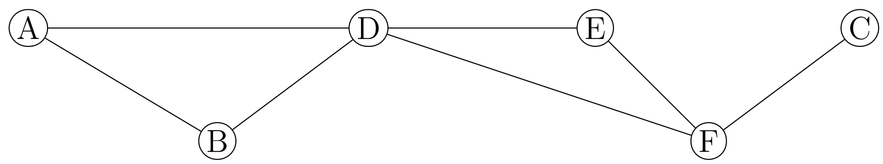
Graph G is at right.
V is the set of nodes in G.
$M = \{0, 1, 2, 3, 4\}$



Define $f : M \to \mathbb{P}(V)$ by $f(n) = \{p \in V \ : \ d(p, F) = n\}$, where $d(a, b)$ is the (shortest-path) distance between a and b. Let $P = \{f(n) \mid n \in M\}$.

(6 points) Fill in the following values:

f(0) =

Solution: $\{F\}$

f(1) =

Solution: $\{C, D, E\}$

P =

Solution: $\{\emptyset, \{F\}, \{C, D, E\}, \{A, B\}\}$

(7 points) Is P a partition of V? For each of the conditions required to be a partition, briefly explain why P does or doesn't satisfy that condition.

Solution: No, P is not a partition of V. The subsets cover all of V with no partial overlap. However, P contains the empty set, since $f(3) = f(4) = \emptyset$.

(2 points) Check the (single) box that best characterizes each item.

Let A be a non-empty set,
$\{A\}$ is a partition of A.

always ☑ sometimes ☐ never ☐

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(9 points) Recall that the symmetric difference of two sets A and B written $A \oplus B$ contains all the elements that are in one of the two sets but not the other. That is $A \oplus B = (A - B) \cup (B - A)$. For any set of integers A, let $[A] = \{B \in \mathbb{P}(\mathbb{Z}) \mid A \oplus B \text{ is finite } \}$

Explain clearly what is in $[\{1, 2, 3\}]$. Also, is it true that $[\mathbb{Z}] = [\mathbb{N}]$? Briefly justify your answer.

Solution:

$[\{1, 2, 3\}]$ contains all finite sets of integers.

$\mathbb{N}$ is in $[\mathbb{N}]$. However, $\mathbb{N}$ can't be in $[\mathbb{Z}]$, since the symmetric difference between $\mathbb{Z}$ and $\mathbb{N}$ includes all the the negative integers and therefore is infinite. So $[\mathbb{Z}]$ cannot be equal to $[\mathbb{N}]$.

(6 points) Check the (single) box that best characterizes each item.

$\binom{0}{0}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ undefined ☐

$\mathbb{P}(A \cup B) = \mathbb{P}(A) \cup \mathbb{P}(B)$ always ☐ sometimes ☑ never ☐

If you want to take 4 classes next semester, out of 35 classes being offered, how many different choices do you have?

$\frac{35!}{31!4!}$ ☑ 35^4 ☐

$\frac{35!}{31!}$ ☐ 4^4 ☐