CS 173, Spring 2016
Examlet 13, Part A

NETID:

FIRST: | **LAST:**

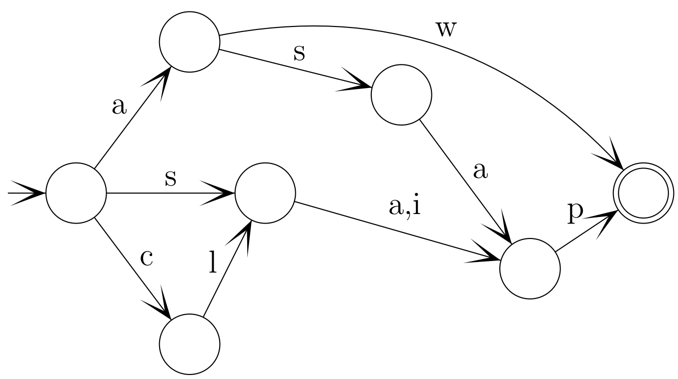
Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

(10 points) Recall that a phone lattice is a state diagram representing sequences of letters. Each edge in a phone lattice has a single letter on it. In a "deterministic" state diagram, if you look at any state s and any letter a, there is never more than one edge labelled a leaving state s.

Draw a deterministic phone lattice representing exactly the following set of words, using no more than 10 states and, if you can, no more than 8.

asap, sip, sap, clip, clap, aw

Solution:



(5 points) Suppose we are making a deterministic phone lattice using a fixed set of n states and a fixed set of p different characters. (Deterministic means that each state has a single outgoing transition for each character.) In how many different ways could we construct a transition function for this lattice?

Solution: The domain for the transition function contains np (state, character) pairs. The co-domain contains n states. So there are n^{np} different transition functions.

CS 173, Spring 2016
Examlet 13, Part B

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(5 points) Suppose that A and B are sets and A is known to be uncountable. Can we conclude that $A \times B$ is uncountable? Briefly justify your answer.

Solution: This is false. Suppose that B is the empty set. Then $A \times B = \emptyset$, which is countable.

(10 points) Check the (single) box that best characterizes each item.

The set of chords (simultaneous combinations of notes) playable on an 88-key piano.	finite [√]	countably infinite [ ]	uncountable [ ]
The set of all polynomials with real coefficients.	finite [ ]	countably infinite [ ]	uncountable [√]
The rational numbers have the same cardinality as the integers.	true [√]	false [ ]	not known [ ]
The set of all (finite, unlabelled) graphs, where isomorphic graphs are treated as the same object.	finite [ ]	countably infinite [√]	uncountable [ ]
$\mathbb{N}^2$ has the same cardinality as $\mathbb{N}$.	true [√]	false [ ]	not known [ ]

CS 173, Spring 2016
Review, Part A

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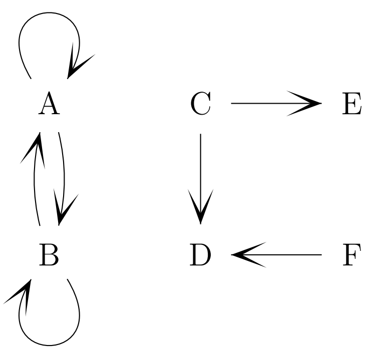
Discussion: **Monday 9 10 11 12 1 2 3 4 5**

(5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: [√]

(10 points) Check the (single) box that best characterizes each item.

$p \rightarrow q \equiv \neg q \rightarrow \neg p$ true [√] false ☐

Two positive integers p and q are relatively prime if and only if $\gcd(p, q) > 1$. true ☐ false [√]

$\sum_{k=1}^{n+1} 2^k$ $2^{n+1} + 1$ ☐ $2^{n+2} - 1$ ☐ $2^{n+2} - 2$ [√] $2^n - 2$ ☐

If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one. true ☐ false [√]

$g : \mathbb{Z} \rightarrow \mathbb{Z}$, $g(x) = |x|$ one-to-one ☐ not one-to-one [√] not a function ☐

CS 173, Spring 2016 Review, Part B

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(5 points) Justin needs to pick 17 toy animals to give to children at a party. The animals come in 5 kinds: dogs, dinosaurs, cows, lizards, and fish. How many different ways can he choose his set of toys?

Solution: $\binom{17+4}{4}$.

(10 points) Check the (single) box that best characterizes each item.

All elements of M are also elements of X.

- $M = X$ ☐
- $M \subseteq X$ ☑ √
- $X \subseteq M$ ☐

The number of edges in the 4-dimensional hypercube Q_4

- 5 ☐
- 12 ☐
- 32 ☑ √
- 64 ☐

The diameter of a full, complete tree of height h.

- $\leq h$ ☐
- h ☐
- $h+1$ ☐
- $2h$ ☑ √
- $\leq 2h$ ☐

W_n has a Euler circuit.

- always ☐
- sometimes ☐
- never ☑ √

Karatsuba's integer multiplication algorithm

- $\Theta(n^2)$ ☐
- $\Theta(n^3)$ ☐
- $\Theta(n \log n)$ ☐
- $\Theta(n^{log_2 3})$ ☑ √
- $\Theta(n^{log_3 2})$ ☐
- $\Theta(2^n)$ ☐