CS 173, Spring 2016
Examlet 13, Part A

NETID:

FIRST: | **LAST:**

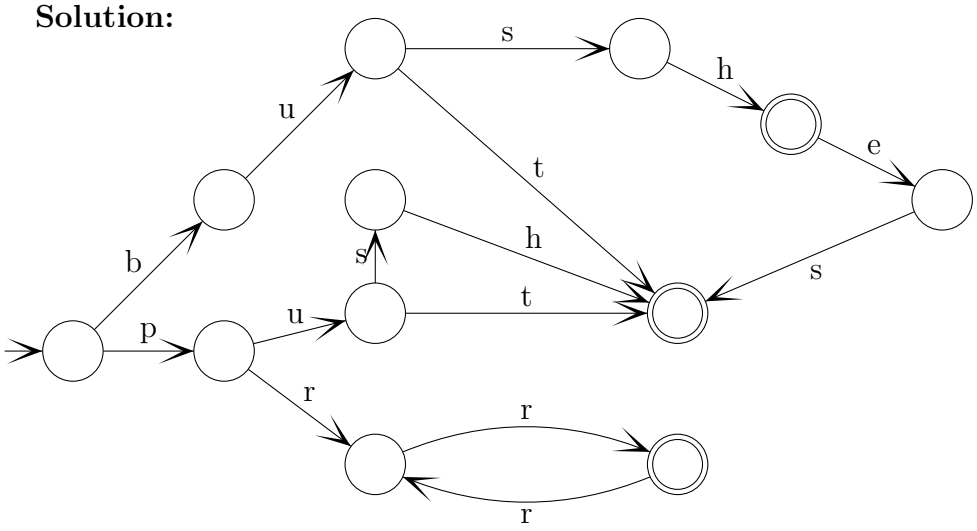
Discussion: Monday 9 10 11 12 1 2 3 4 5

(15 points) Recall that a phone lattice is a state diagram representing sequences of letters. Each edge in a phone lattice has a single letter on it. In a "deterministic" state diagram, if you look at any state s and any letter a, there is never more than one edge labelled a leaving state s.

Draw a deterministic phone lattice representing exactly the following set of words, using no more than 16 states and, if you can, no more than 13.

put, push, but, bush, bushes,
prr, prrrr, prrrrrr .... [i.e. p followed a non-zero, even number of r's]

Solution:



CS 173, Spring 2016
Examlet 13, Part B

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Discussion: Monday 9 10 11 12 1 2 3 4 5

(5 points) A black and white digitized picture is a (finite) 2D array of integer values between 0 and 255. A color digitized picture consists of three such arrays (red, green, and blue). Is the set of all possible color digitzed pictures countable or uncountable?

Solution: It's countable. If you fix a specific area (i.e. number of values in the array), there are only a finite number of digitized pictures with that area. So the full set of color digitized pictures is a countable union of finite sets, which is countable.

(10 points) Check the (single) box that best characterizes each item.

The set of (unlabelled, finite) binary trees with exactly 4 leaves.
finite ☐ countably infinite ☑ uncountable ☐

$|A \times A| > |A|$
true ☐ false ☐ true for some sets ☑

There exist mathematical functions that cannot be computed by any C program.
true ☑ false ☐ not known ☐

The real numbers
finite ☐ countably infinite ☐ uncountable ☑

$\mathbb{P}(\mathbb{Z})$
finite ☐ countably infinite ☐ uncountable ☑

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Review, Part A

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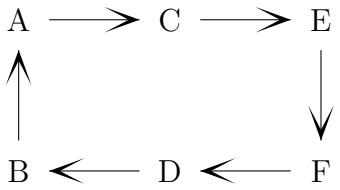
(5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☑

Symmetric: ☐ Antisymmetric: ☑

Transitive: ☐

(10 points) Check the (single) box that best characterizes each item.

$p \vee q \equiv \neg p \to q$ true ☑ false ☐

For all prime numbers p, there are exactly two natural numbers q such that $q \mid p$. true ☑ false ☐

$\sum_{k=0}^{n+1} 2^k$ $2^{n+1}+1$ ☐ $2^{n+2}-1$ ☑ $2^{n+2}-2$ ☐ $2^{n+1}-1$ ☐

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the real numbers is the ____ of f. domain ☐ co-domain ☑ image ☐

$g : \mathbb{N} \to \mathbb{Z}$, $g(x) = x$ onto ☐ not onto ☑ not a function ☐

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(5 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is such that $f(n) = n!$. Give a recursive definition of f

Solution:

$f(0) = 1$, and $f(n) = nf(n-1)$ for $n \geq 1$.

You could also have used $f(n+1) = (n+1)f(n)$ for $n \geq 0$.

(10 points) Check the (single) box that best characterizes each item.

Chromatic number of a bipartite graph with at least one edge: 1 ☐ 2 ☑ 3 ☐ can't tell ☐

Number of edges in $K_{3,4}$: 7 ☐ 12 ☑ 14 ☐ 49 ☐

A tree with n nodes has: n edges ☐ $n-1$ edges ☑ $\leq n$ edges ☐ $n/2$ edges ☐ $\log n$ edges ☐

$T(1) = d$
$T(n) = 3T(n/3) + c$: $\Theta(\log n)$ ☐ $\Theta(n)$ ☑ $\Theta(n \log n)$ ☐ $\Theta(n^2)$ ☐

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is: an integer ☐ a set of integers ☑ one or more integers ☐ a power set ☐