CS 173, Spring 2016 Examlet 13, Part A

NETID:

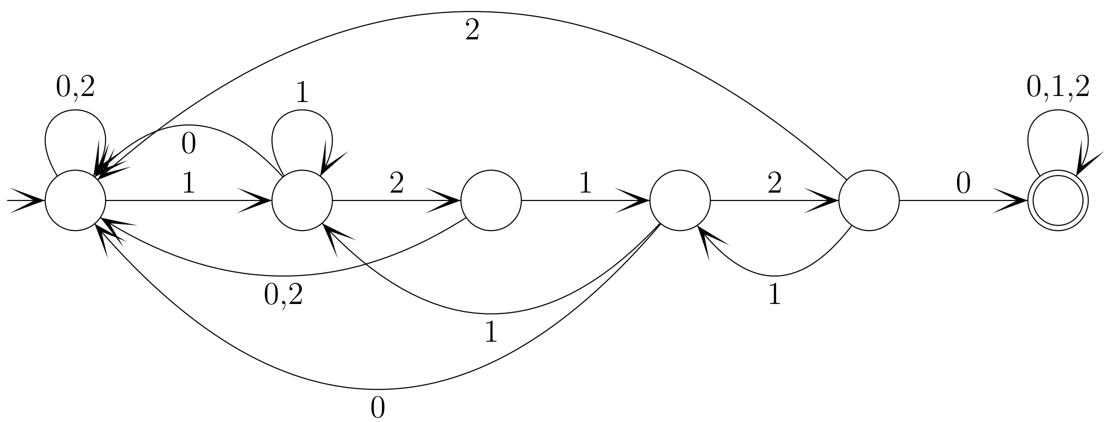
FIRST: **LAST:**

Discussion: **Monday 9 10 11 12 1 2 3 4 5**

(15 points) Professor Martinez needs a state machine that will recognize the sequence 12120 when typed on a keypad. Specifically, it must read any sequence of the digits 0, 1, and 2. It should move into a final state immediately after seeing 12120, and then remain in that final state as further characters come in. For efficiency, the state machine must be deterministic, i.e. if you look at any state s and any action a, there is never more than one edge labelled a leaving state s.

Draw a deterministic state diagram that will meet his needs, using no more than 9 states and, if you can, no more than 6.

Solution:



CS 173, Spring 2016
Examlet 13, Part B

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Discussion: **Monday** **9** **10** **11** **12** **1** **2** **3** **4** **5**

(5 points) A Stark chain consists of the number line with an integer value between 1 and 67 placed at each integer position. For example, we can create one specific Stark chain by placing 1 at zero, 3 on each negative integer, 23 on each even positive integer, and 67 on each odd positive integer. Is the set of all Stark chains countable or uncountable? Briefly justify your answer.

Solution: The set of all Stark chains is uncountable. Each Stark chain can be viewed as a function from $\mathbb{Z}$ to the set $\{1, 2, \ldots, 67\}$. The set of functions from any infinite set to any set with at least 2 elements is uncountable.

(10 points) Check the (single) box that best characterizes each item.

Every mathematical function $f : \mathbb{N} \to \mathbb{N}$ has a corresponding C++ program that will compute $f(n)$ given an input of n.	true ☐	false ☑ √	not known ☐
The rational numbers have the same cardinality as the reals.	true ☐	false ☑ √	not known ☐
$\mathbb{R} - \mathbb{Q}$	finite ☐	countably infinite ☐	uncountable ☑ √
The set of 10-digit US phone numbers.	finite ☑ √	countably infinite ☐	uncountable ☐
If $f : A \to B$ is one-to one	$\lvert A\rvert < \lvert B\rvert$ ☐	$\lvert A\rvert \leq \lvert B\rvert$ ☑ √	$\lvert A\rvert = \lvert B\rvert$ ☐

CS 173, Spring 2016
Review, Part A

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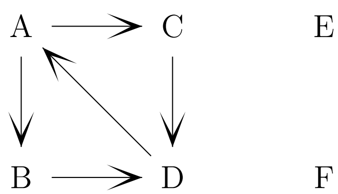
(5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: [ ] Irreflexive: [√]

Symmetric: [ ] Antisymmetric: [√]

Transitive: [ ]

(10 points) Check the (single) box that best characterizes each item.

$p \wedge q \equiv \neg(p \rightarrow \neg q)$ true [√] false [ ]

Zero is a multiple of 7. true [√] false [ ]

$\sum_{i=1}^{p-1} i$ $\frac{p(p-1)}{2}$ [√] $\frac{(p-1)^2}{2}$ [ ] $\frac{p(p+1)}{2}$ [ ] $\frac{(p-1)(p+1)}{2}$ [ ]

Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, there are two vertices with the same color. true [√] false [ ]

$g : \mathbb{Z} \rightarrow \mathbb{Z}$,
$g(x) = |x|$ one-to-one [ ] not one-to-one [√] not a function [ ]

CS 173, Spring 2016 Review, Part B

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(5 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for f to be $O(g)$.

Solution: There are positive reals c and k such that $0 \le f(x) \le cg(x)$ for every $x \ge k$.

(10 points) Check the (single) box that best characterizes each item.

Chromatic number of G	$\mathcal{C}(G)$ ☐	$\phi(G)$ ☐	$\chi(G)$ ☑ √	$\parallel G \parallel$ ☐
The Travelling Salesman problem can be solved in polynomial time.	true ☐	false ☐	not known ☑ √	
A tree node is an ancestor of itself.	always ☑ √	sometimes ☐	never ☐	
$T(1) = c$ $T(n) = 2T(n/2) + n$	$\Theta(n)$ ☐	$\Theta(n^2)$ ☐	$\Theta(n \log n)$ ☑ √	$\Theta(2^n)$ ☐
Number of connected components in W_7.	1 ☑ √	7 ☐	8 ☐	14 ☐