CS 173, Fall 17 Examlet 2, Part A

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(15 points) Prove the following claim, using direct proof and your best mathematical style.

For any integers x, y, and z, if $100x+10y+z$ is divisible by 9, then $x+y+z$ is divisible by 9.

Hint: analyze the difference between $100x + 10y + z$ and $x + y + z$.

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(15 points) Prove the following claim, working directly from the definitions of "remainder" and "divides", and using your best mathematical style.

For all real numbers k, m, n and r $(n \neq 0)$, if $r = \text{remainder}(m, n)$, $k \mid m$, and $k \mid n$, then $k \mid r$.

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(15 points) Recall that a real number p is rational if there are integers m and n (n non-zero) such that $p = \frac{m}{n}$. Use this definition and your best mathematical style to prove the following claim:

For all real numbers x and y, $x \neq 0$, if x and $\frac{y+1}{2}$ are rational, then $\frac{5}{x} + y$ is rational.

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(15 points) Recall that $\gcd(m,n)$ is the largest integer that divides both m and n. Use this definition, the definition of divides, and your best mathematical style to prove the following claim by contrapositive.

For all integers p and q, if $p+6q=23$ then $\gcd(p,q)\neq 7$.

You must begin by explicitly stating the contrapositive of the claim:

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(15 points) An integer k is a perfect square if $k = n^2$ where n is a non-negative integer. Prove the following claim:

For any integer p, if $p \geq 8$ and $p + 1$ is a perfect square, then p is composite (aka not prime).

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(15 points) Prove the following claim, using your best mathematical style and the following definition of congruence mod k: $a \equiv b \pmod{k}$ if and only if $a = b + nk$ for some integer n.

Claim: For all integers a, b, c, d, and k (k positive), if $a \equiv b \pmod{k}$ and $c \equiv d \pmod{k}$ then $a^2 + c \equiv b^2 + d \pmod{k}$.