CS 173, Fall 17 Examlet 2, Part B

Name:____

NetID:________________________ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all non-zero integers a and b, if $a \mid b$ and $b \mid a$, then $a = b$.

   Solution: This is false. Consider $a = 3$ and $b = -3$. Then $a \mid b$ and $b \mid a$, but $a \neq b$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2015, 837)$. Show your work.

   Solution:

   $2015 - 837 \times 2 = 2015 - 1674 = 341$

   $837 - 341 \times 2 = 837 - 682 = 155$

   $341 - 155 \times 2 = 341 - 310 = 31$

   $155 - 31 \times 5 = 0$

   So the GCD is 31.

3. (4 points) Check the (single) box that best characterizes each item.

   $0 \mid 0$ true [√] false [ ]

   Two positive integers p and q are relatively prime if and only if $\gcd(p, q) > 1$. true [ ] false [√]

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, b) = n$ and $\gcd(a, c) = p$, then $\gcd(a, bc) = np$.

   Solution: This is false. Consider $a = b = c = 3$. Then if $\gcd(a, b) = 3$ and $\gcd(a, c) = 3$, but $\gcd(a, bc)$ is 3, not 9.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1609, 563)$. Show your work.

   Solution:

   $1609 - 2 \times 563 = 1609 - 1126 = 483$

   $563 - 483 = 80$

   $483 - 6 \times 80 = 3$

   $80 - 26 \times 3 = 80 - 78 = 2$

   $3 - 2 = 1$

   $2 - 2 \times 1 = 0$

   So the GCD is 1.

3. (4 points) Check the (single) box that best characterizes each item.

   For any positive integers p and q, if $\text{lcm}(p, q) = pq$, then p and q are relatively prime. true [√] false [ ]

   $(5 \times 5) \equiv 1 \pmod 6$ true [√] false [ ]

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, $\gcd(ca, cb) = c \cdot \gcd(a, b)$

   Solution: This is true.

   c divides both ca and cb. So $\gcd(ca, cb)$ must have the form cm, where m is an integer. But then cm is the largest integer that divides both ca and cb if and only if m is the largest integer that divides both a and b.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2380, 391)$. Show your work.

   Solution:

   $2380 - 391 \times 6 = 2380 - 2346 = 34$

   $391 - 34 \times 11 = 391 - 374 = 17$

   $34 - 17 \times 2 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

$25 \equiv 4 \pmod 7$	true [√]	false [ ]
Two positive integers p and q are relatively prime if and only if $\gcd(p, q) = 1$.	true [√]	false [ ]

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers a, b, and c, if $a \mid bc$, then $a \mid b$ or $a \mid c$

   Solution: This is false. Consider $a = 6$, $b = 3$, $c = 2$. Then $a|bc$, but a doesn't divide either b or c.

2. (6 points) Write pseudocode (iterative or recursive) for a function gcd(a,b) that implements the Euclidean algorithm. Assume both inputs are positive.

   Solution:

   ```
   gcd(a,b)

       x=a
       y=b
       while (b > 0)
           r = remainder(a,b)
           a = b
           b = r
       return a
   ```

3. (4 points) Check the (single) box that best characterizes each item.

$2 \mid -4$	true ☑	false ☐
If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(a, b) = \gcd(r, a)$	true ☐	false ☑

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1. (5 points) Explain how to use the Euclidean algorithm to test whether two positive integers p and q are relatively prime.

   Solution: Use the Euclidean algorithm to compute $\gcd(p, q)$. If the output is 1, p, and q are relatively prime.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1702, 1221)$. Show your work.

   Solution: $1702 - 1221 = 481$

   $1221 - 481 \times 2 = 1221 - 962 = 259$

   $481 - 259 = 222$

   $259 - 222 = 37$

   $222 - 6 \times 37 = 0$

   So $\gcd(1702, 1221) = 37$

3. (4 points) Check the (single) box that best characterizes each item.

If p, q, and k are primes, then $gcd(pq, qk) =$	q ☐	pq ☐	pqk ☐	$q \gcd(p, k)$ ☑

$29 \equiv 2 \pmod 9$	true ☑	false ☐

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1. (5 points) For any real numbers x and y, let's define the operation $\oslash$ by the equation $x \oslash y = x^2 + y^2$. Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For any real numbers x,y, and z, $(x \oslash y) \oslash z = x \oslash (y \oslash z)$

   Solution: This is not true. Consider $x = y = 1$ and $z = 2$. Then $(x \oslash y) \oslash z = (x^2 + y^2)^2 + z^2 = (1+1)^2 + 2^2 = 8$. But $x \oslash (y \oslash z) = x^2 + (y^2 + z^2)^2 = 1^2 + (1^2 + 2^2)^2 = 1 + 5^2 = 26$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(2737, 2040)$. Show your work.

   Solution:

   $2737 - 2040 = 697$

   $2040 - 697 \times 2 = 2040 - 1394 = 646$

   $697 - 646 = 51$

   $646 - 51 \times 12 = 646 - 612 = 34$

   $51 - 34 = 17$

   $34 - 17 = 0$

   So the GCD is 17.

3. (4 points) Check the (single) box that best characterizes each item.

   $\gcd(p, q) = \frac{pq}{\text{lcm}(p, q)}$ (p and q positive integers) always [√] sometimes [ ] never [ ]

   $-3 \equiv 3 \pmod 4$ true [ ] false [√]