CS 173, Fall 17 Examlet 3, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

$A = \{(x, y, z) \in \mathbb{R}^3 \mid 0 < x < y - 1\}$

$B = \{(a, b, c) \in \mathbb{R}^3 \mid b^2 + 2 < c^2\}$

$C = \{(p, q, r) \in \mathbb{R}^3 \mid p^2 < r^2\}$

Prove that $A \cap B \subseteq C$.

Solution: Let $(x, y, z) \in A \cap B$. Then $(x, y, z) \in A$, so (x, y, z) is a triple of real numbers with $0 < x < y - 1$. Also $(x, y, z) \in B$, so $y^2 + 2 < z^2$.

We know that $0 < x < y - 1$. Since $y - 1 > 0$, $y > 0$, so $-2y < 0$. Squaring both sides of $x < y - 1$ and using the fact that both sides of the equation are positive, we get $x^2 < y^2 - 2y + 1$. So $x^2 < y^2 + 1 < y^2 + 2$. But we know that $y^2 + 2 < z^2$. So we have $x^2 < z^2$, and therefore $(x, y, z) \in C$, which is what we needed to show.

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$A = \{(x, y) \in \mathbb{R}^2 \mid x = \lfloor 3y + 5 \rfloor \}$

$B = \{(p, q) \in \mathbb{Z}^2 \mid 2p + q \equiv 3 \pmod 7 \}$

Prove that $A \cap \mathbb{Z}^2 \subseteq B$.

Use the following definition of congruence mod k: if s, t, k are integers, k positive, then $s \equiv t \pmod k$ if and only if $s = t + nk$ for some integer n.

Solution: Let (x, y) be an element of $A \cap \mathbb{Z}^2$. Then (x, y) is an element of A and, also, both x and y are integers.

By the definition of Z, $x = \lfloor 3y + 5 \rfloor$. Since y is an integer, $3y + 5$ must also be an integer. So $\lfloor 3y + 5 \rfloor = 3y + 5$. Therefore, $x = 3y + 5$.

Now, consider $2x + y$.

$$2x + y = 2(3y + 5) + y = 7y + 10 = 7(y + 1) + 3$$

$y + 1$ is an integer, since y is an integer. So this means that $2x + y \equiv 3 \pmod 7$. Therefore, (x, y) is an element of B, which is what we needed to show.

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Name:____

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$A = \{(x, y) \in \mathbb{R}^2 \mid y = x^2 - 3x + 2\}$

$B = \{(p, q) \in \mathbb{R}^2 \mid p \geq 0\}$

$C = \{(m, n) \in \mathbb{R}^2 \mid n \geq 1\}$

Prove that $A \subseteq B \cup C$.

Solution:

Suppose that (x, y) is an element of A. Then $y = x^2 - 3x + 2 = (x - 1)(x - 2)$. There are two cases:

Case 1: $x \geq 0$. Then $(x, y) \in B$ so $(x, y) \in B \cup C$.

Case 2: $x \leq 0$. Then $x - 1 \leq -1$ and $x - 2 \leq -2$. So $y = (x - 1)(x - 2) \geq 2 \geq 1$. So $(x, y) \in C$. And therefore $(x, y) \in B \cup C$.

In both cases $(x, y) \in B \cup C$, which is what we needed to show.

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$A = \{(x, y) \in \mathbb{R}^2 \ : \ |x + y + 2| < 5\}$

$B = \{(p, q) \in \mathbb{R}^2 \ : \ |p - q| < 3\}$

$C = \{(a, b) \in \mathbb{R}^2 \ : \ |ab| < 10\}$

Prove that $A \cap B \subseteq C$.

Solution: This claim isn't true as stated. It's ok if you did any of the following:

- An almost-right proof, with a gap or a dodgy step (e.g. involving the inequalities).
- A proof of a slightly modified version of the claim, correct or almost-right.
- A well-explained counter-example.

For the proofs, we'll be looking at whether your basic style and outline are sensible, even if the algebra didn't or couldn't work.

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$A = \{(x, y) \in \mathbb{Z}^2 \mid 2xy + 6y - 5x - 15 \geq 0\}$

$B = \{(a, b) \in \mathbb{Z}^2 \mid a \geq 0\}$

$C = \{(p, q) \in \mathbb{Z}^2 \mid q \geq 0\}$

Prove that $(A \cap B) \subseteq C$.

Solution: Suppose that (x, y) is an element of $(A \cap B)$. This means that (x, y) is an element of A and (x, y) is an element of B. So $2xy + 6y - 5x - 15 \geq 0$ and $x \geq 0$, by the definitions of A and B.

Notice that $2xy + 6y - 5x - 15 = (x + 3)(2y - 5)$. So $(x + 3)(2y - 5) \geq 0$. We know that $x + 3$ is positive because $x \geq 0$. So we must have $(2y - 5) \geq 0$.

Now, if $(2y - 5) \geq 0$, then $2y \geq 5$. So $y \geq \frac{5}{2}$. So $y \geq 0$. This means that (x, y) is an element of C which is what we needed to show.

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$A = \{(x, y, z) \in \mathbb{R}^3 \ : \ |x + y + z| = 20\}$

$B = \{(a, b, c) \in \mathbb{N}^3 \ : \ a + b < 5\}$

$C = \{(p, q, r) \in \mathbb{R}^3 \ : \ r > 10\}$

Prove that $A \cap B \subseteq C$.

Solution: Let $(x, y, z) \in A \cap B$. Then $(x, y, z) \in A$, so $|x + y + z| = 20$. Also $(x, y, z) \in B$, so $x + y < 5$ and x, y, and z are all natural numbers.

Since x, y, and z are natural numbers, they can't be negative. So $x + y + z$ isn't negative. Therefore $x + y + z = |x + y + z| = 20$. So $z = 20 - (x + y)$.

Since $z = 20 - (x + y)$ and $x + y < 5$, $z > 15$. So $z > 10$, which means that $(x, y, z) \in C$. This is what we needed to show.