CS 173, Fall 17 Examlet 3, Part B

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (4 points) $A = \{\text{oak}, \text{apple}, \text{maple}, \text{elm}\}$ $B = \{\text{tree}, \text{oak}, \emptyset\}$

   $(A \times \emptyset) \cap B =$

   Solution: $A \times \emptyset = \emptyset$ So $(A \times \emptyset) \cap B = \emptyset \cap B = \emptyset$

   $\{\frac{p}{q} \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} =$

   Solution: $\{\frac{p}{q} \ : \ p \in \mathbb{Z}^+, q \in \mathbb{Z}^+, \text{ and } pq = 6\} = \{\frac{1}{6}, \frac{2}{3}, \frac{3}{2}, 6\}$

2. (4 points) Check the (single) box that best characterizes each item.

   For all positive integers n, if $n! < -10$, then $n > 8$.
   true [√] false [ ] undefined [ ]

   Let A and B be disjoint. $|A - B| = |A| - |B|$
   true for all sets A and B [ ] true for some sets A and B [√]
   false for all sets A and B [ ]

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{41}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

   Solution: $[3]^2 = [9] = [2]$

   $[3]^4 = [2]^2 = [4]$

   $[3]^8 = [4]^2 = [16] = [2]$

   $[3]^{16} = [2]^2 = [4]$

   $[3]^{32} = [4]^2 = [2]$

   $[3]^{41} = [3]^{32} \cdot [3]^8 \cdot [3] = [2][2][3] = [12] = [5]$

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A and B, $(A - B) \cup (B - A) \subseteq (A \cup B) - (A \cap B)$

   Solution: This is true. An element of $(A - B) \cup (B - A)$ must be in exactly one of the two sets. So it must be in $(A \cup B)$ but not in $(A \cap B)$.

2. (4 points) Check the (single) box that best characterizes each item.

$A \times A = A$ (Assume $A \neq \emptyset$)	true for all sets A	☐	false for all sets A	☑
	true for some sets A	☐		

$\{1, 2\} \times \emptyset =$	$\emptyset$	☑	$\{(1, \emptyset), (2, \emptyset)\}$	☐	$\{1, 2, \emptyset\}$	☐
	$\{\emptyset\}$	☐	$\{1, 2\}$	☐	undefined	☐

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{32} = [4]^2 = [16] = [7]$

   $[5]^{38} = [5]^{32} \cdot [5]^4 \cdot [5]^2 = [7] \cdot [4] \cdot [7] = [28] \cdot [7] = [1] \cdot [7] = [7]$

CS 173, Fall 17 Examlet 3, Part B

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $(A - B) - C \subseteq A - C$

   Solution: This is true. Suppose that x is in $(A - B) - C$. Then x must be in A, but not in B or C. Since x is in A but not in C, x is in $A - C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $|A - B| = |A| - |B|$
   - true for all sets A and B ☐
   - true for some sets A and B ☑ ($\surd$)
   - false for all sets A and B ☐

   For all reals n, if $n^2 = 101$, then $n > 11$.
   - true ☐
   - false ☑ ($\surd$)
   - undefined ☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = ([7]^2)^2 = [-3]^2 = [9]$

   $[7]^8 = ([7]^4)^2 = [9]^2 = [81] = [3]$

   $[7]^{16} = ([7]^8)^2 = [3]^2 = [9]$

   $[7]^{21} = [7]^{16} \cdot [7]^4 \cdot [7] = [9] \cdot [9] \cdot [7] = [81] \cdot [7] = [3] \cdot [7] = [21] = [8]$

CS 173, Fall 17 Examlet 3, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (4 points) $A = \{\text{earth}, \text{air}, \text{fire}\}$ $B = \{ (\text{fire}, 3), (\text{water}, 2) \}$ $C = \{ 1, 2, 3 \}$

   $(A \times C) \cap B =$

   Solution: $\{ (\text{fire}, 3)\}$

   $\{p + q \mid p \in \mathbb{Z}, q \in \mathbb{Z}, pq = 6\} =$

   Solution: $\{7, -7, 5, -5\}$

2. (4 points) Check the (single) box that best characterizes each item.

   $A = \overline{A}$ (Assume the universe is not empty.)

true for all sets A	☐	true for some sets A	☐
false for all sets A	☑ √		

   $\{1, 2\} \times \{\emptyset\} =$

$\emptyset$	☐	$\{(1, \emptyset), (2, \emptyset)\}$	☑ √	$\{1, 2, \emptyset\}$	☐
$\{\emptyset\}$	☐	$\{1, 2\}$	☐	undefined	☐

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{41}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{32} = [4]^2 = [16] = [7]$

   $[5]^{41} = [5]^{32} \cdot [5]^8 \cdot [5] = [7] \cdot [7] \cdot [5] = [49] \cdot [5] = [4] \cdot [5] = [20] = [2]$

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, if $A \times C \subseteq B \times C$, then $A \subseteq B$.

   Solution: This is false. Suppose that $A = \{1, 2\}$, $B = \{10, 11\}$, and $C = \emptyset$. Then $A \times C = \emptyset = B \times C$, so $A \times C \subseteq B \times C$. But $A \not\subseteq B$.

2. (4 points) Check the (single) box that best characterizes each item.

   $\forall x \in \mathbb{N}$, if $x^2 < -3$, then $x > 1000$. true [√] false [ ] undefined [ ]

   $A \cap B \subseteq A$ true for all sets A and B [√] true for some sets A and B [ ] false for all sets A and B [ ]

3. (7 points) In $\mathbb{Z}_{17}$, find the value of $[5]^{42}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \le n \le 16$.

   Solution:

   $[5]^2 = [25] = [8]$

   $[5]^4 = [8]^2 = [64] = [-4]$

   $[5]^8 = [-4]^2 = [16] = [-1]$

   $[5]^{16} = [-1]^2 = [1]$

   $[5]^{32} = [1]^2 = [1]$

   So

   $[5]^{42} = [5]^{32} \cdot [5]^8 \cdot [5]^2 = [1][-1][8] = [-8] = [9]$

CS 173, Fall 17 Examlet 3, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (4 points) State the Inclusion Exclusion Principle/Formula for two sets.

   Solution: For any sets A and B, $|A \cup B| = |A| + |B| - |A \cap B|$

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset \times A = A \times \emptyset$
   - true for all sets A ☑ (√)
   - false for all sets A ☐
   - true for some sets A ☐

   $A \cap B = A \cup B$
   - true for all sets A and B ☐
   - true for some sets A and B ☑ (√)
   - false for all sets A and B ☐

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{19}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10]$

   $[7]^4 = [100] = [9]$

   $[7]^8 = [9]^2 = [81] = [3]$

   $[7]^{16} = [3]^2 = [9]$

   $[7]^{19} = [7]^{16} \cdot [7]^[2] \cdot [7] = [9] \cdot [10] \cdot [7]$

   $[9] \cdot [10] \cdot [7] = [90] \cdot [7] = [-1] \cdot [7] = [-7] = [6]$

   So $[7]^{19} = [6]$