CS 173, Fall 17 **Examlet 4, Part A** **Name:**_____

NetID:_____ **Lecture:** A   B

Discussion: **Thursday** **Friday** 9   10   11   12   1   2   3   4   5   6

Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(a, b)T(p, q)$ if and only if $ab \mid p$

Working directly from the definition of divides, prove that T is transitive.

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Let's define a relation R on $\mathbb{Z}^3$ as follows:

$(a, b, c)R(x, y, z)$ if and only if $c = x$, $a = y$, and $b = z$.

Working directly from this definition, prove that R is antisymmetric.

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Define the relation $\sim$ on $\mathbb{Z}$ by

$x \sim y$ if and only if $5 \mid (3x + 7y)$

Working directly from the definition of divides, prove that $\sim$ is transitive.

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Let's define the relation $\succeq$ on $\mathbb{N}^2$ by

$(x, y) \succeq (a, b)$ if and only if $x - a \geq 2$ and $y \geq b$.

Prove that $\succeq$ is transitive.

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A closed interval of the real line can be represented as a pair (c, r), where c is the center of the interval and r is its radius. Let $X = \{(c, r) \mid c, r \in \mathbb{R}, r \geq 0\}$ be the set of closed intervals represented this way.

Now, let's define the interval containment $\preceq$ on X as follows

$\quad (c, r) \preceq (d, q)$ if and only if $r \leq q$ and $|c - d| + r \leq q$.

Prove that $\preceq$ is antisymmetric.

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Let $A = \mathbb{N} \times \mathbb{N}$, i.e. pairs of natural numbers.

Define a relation $\gg$ on A as follows:

$(x, y) \gg (p, q)$ if and only if there exists an integer $n \geq 1$ such that $(x, y) = (np, nq)$.

Prove that $\gg$ is transitive.