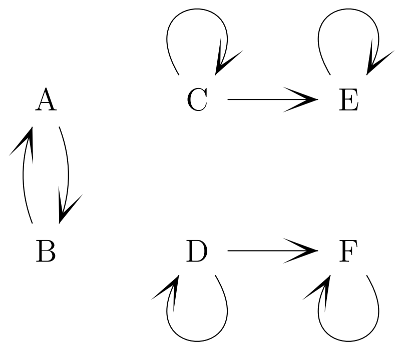
CS 173, Fall 17 Examlet 4, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☐

(That is, no boxes checked.)

2. (5 points) Let $\sim$ be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y) \sim (p, q)$ if and only if $x^2 + y^2 = p^2 + q^2$. Find three elements in the equivalence class $[(0, 1)]$

Solution:

$(0, 1)$, $(1, 0)$), $(-1, 0)$ (for example)

3. (5 points) Suppose that $\preceq$ is the relation between subsets of the integers such that $A \preceq B$ if and only if $A - B = \emptyset$. (A and B are sets of integers, so $A - B$ is a set difference.) Is $\preceq$ antisymmetric? Informally explain why it's true (e.g. use a Venn diagram) or give a concrete counter-example.

Solution: $\preceq$ is antisymmetric. Suppose that $X - Y = \emptyset$ and $Y - X = \emptyset$.

Notice that $X = (X \cap Y) \cup (X - Y)$. Draw a Venn diagram if this isn't clear. Since $X - Y = \emptyset$, we have $X = (X \cap Y)$.

Similarly, $Y - X = \emptyset$ implies that $Y = (X \cap Y)$.

So $X = X \cap Y = Y$.

[An annotated Venn diagram would work fine as an informal explanation.]

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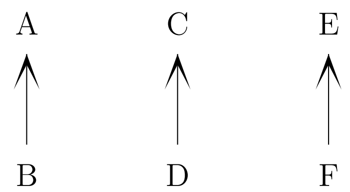

Reflexive:	☐	Irreflexive:	☑ $\surd$
Symmetric:	☐	Antisymmetric:	☑ $\surd$
Transitive:	☑ $\surd$		

2. (5 points) A relation is a partial order if it has which three properties? (Naming the properties is sufficient. You don't have to define them.)

   Solution: reflexive, antisymmetric, transitive

3. (5 points) Suppose that T is the relation on the set of integers such that aTb if and only if $\gcd(a, b) = 3$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not transitive. Suppose $a = 6$, $b = 15$, and $c = 12$. Then $\gcd(a, b) = 3$ and $\gcd(b, c) = 3$, but $\gcd(a, c) = 6$.

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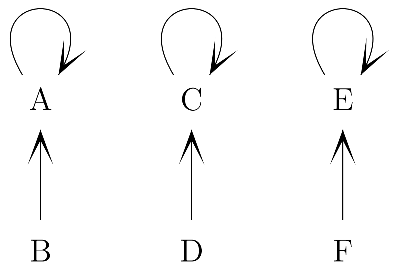

Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☑ √

Transitive: ☑ √

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be symmetric.

   Solution: For any $x, y \in A$, if xRy, then yRx.

3. (5 points) Suppose that R is the relation on $\mathbb{Z}^4$ such that $(a, b, c, d)R(w, x, y, z)$ if and only if $c = w$, $d = x$, $a = y$, and $b = z$. Is R symmetric? Informally explain why it's true or give a concrete counter-example.

   Solution: R is symmetric Suppose we have $(a, b, c, d)R(w, x, y, z)$. Then $c = w$, $d = x$, $a = y$, and $b = z$. Rewriting these equations gives us $y = a$, $z = b$, $w = c$, and $x = d$. This means that $(w, x, y, z)R(a, b, c, d)$.

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

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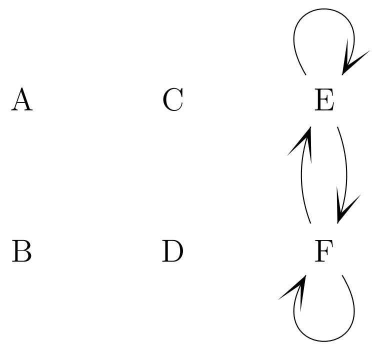

Reflexive: ☐ Irreflexive: ☐

Symmetric: ☑ Antisymmetric: ☐

Transitive: ☑

2. (5 points) Let R be the relation on the integers such that aRb if and only if $2a \equiv -3b \pmod 5$. Find three elements in the equivalence class [7].

   Solution:

   -3, 2, 7 (for example)

3. (5 points) Suppose that R is the relation on $\mathbb{Z}^3$ such that $(a, b, c)R(x, y, z)$ if and only if $c = x$, $a = y$, and $b = z$. Is R transitive? Informally explain why it's true or give a concrete counter-example.

   Solution: R is not transitive. We have

$$(1, 2, 0)R(0, 1, 2) \text{ and } (0, 1, 2)R(2, 0, 1)$$

   but not

$$(1, 2, 0)R(2, 0, 1)$$

   .

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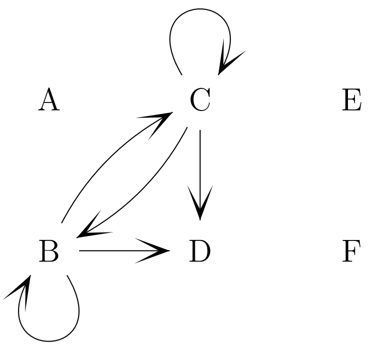

Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: ☑ √

2. (5 points) Suppose that S is the set of all binary strings (i.e. finite sequences of 1's and 0's). Suppose that $\sim$ is the relation on S where $a \sim b$ if and only if a and b are the same length. For example, `01011` $\sim$ `00010`. List three members of [`1111`].

   Solution: For example, `0101`, `1101`, and `0000`.

3. (5 points) Let T be the relation on $\mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $(x, y) = \alpha(p, q)$ for some real number α. Is T symmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: T is not symmetric. We have $(0, 0)T(p, q)$ by setting α to zero but not $(3, 4)T(0, 0)$.

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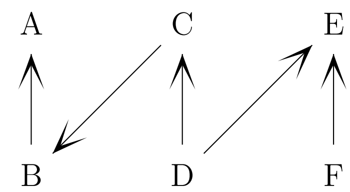

Reflexive:	☐	Irreflexive:	☑ $\surd$
Symmetric:	☐	Antisymmetric:	☑ $\surd$
Transitive:	☐		

2. (5 points) Can a relation with at least one related pair (i.e. at least one arrow in a diagram) be irreflexive, symmetric, and also transitive? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: No, this is not possible. Suppose R is our relation and let x and y be two elements such that xRy. Then yRx because it's symmetric. Then xRx because it's transitive. But xRx means that R can't be irreflexive.

3. (5 points) Suppose that $\succeq$ is the relation between subsets of the integers such that $A \succeq B$ if and only if $A - B \neq \emptyset$. (A and B are sets of integers, so $A - B$ is a set difference.) Is $\succeq$ transitive? Informally explain why it's true or give a concrete counter-example.

   Solution: $\succeq$ is not transitive. Consider $A = C = \{3\}$ and $B = \{4\}$. Then $A - B = \{3\}$ and $B - C = \{4\}$. So $A \succeq B$ and $B \succeq C$. But $A - C = \emptyset$, so we don't have $A \succeq C$.