CS 173, Fall 17 Examlet 5, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is onto. Recall that the max function returns the larger of its two inputs. Let's define $g : \mathbb{N}^2 \to \mathbb{N}$ by $g(x, y) = f(\max(x - 7, 0)) + f(y)$. Prove that g is onto.

2. (5 points) Recall that $\mathbb{Z}^+$ is the set of positive integers. Let's define the function $h : (\mathbb{Z}^+)^2 \to \mathbb{R}$ such that $h(d, e) = 2^d + \frac{1}{e}$. Is h one-to-one? Briefly justify your answer.

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1. (10 points) Let's define the function $f : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $f(x, y) = (x + y, 2x - 3y)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ and $g : \mathbb{Z} \to \mathbb{Z}$ are functions. Let's define the function $f + g$ by $(f + g)(x) = f(x) + g(x)$. Adele claims that if f and g are onto, then $f + g$ is onto. Is this correct? Briefly explain why it is or give a counter-example.

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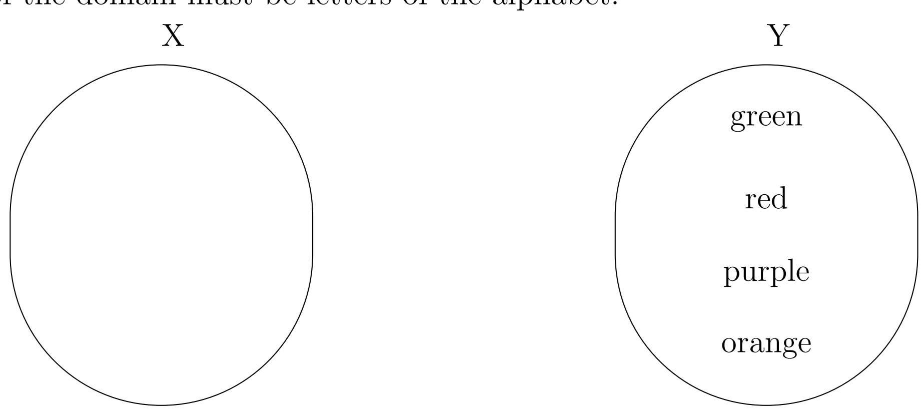
1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $g : \mathbb{Z} \to \mathbb{Z}^2$ by $g(n) = (|n|, f(n)|n|)$. Prove that g is one-to-one.

2. (5 points) Complete this picture to make an example of a function that is one-to-one but not onto, by adding elements to the domain and arrows showing how input values map to output values. The elements of the domain must be letters of the alphabet.



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1. (10 points) If a is any real number, (a, ∞) is the set of all real numbers greater than a. Let's define the function $f : (0, \infty) \to (\frac{5}{4}, \infty)$ by $f(x) = \frac{5x^2+3}{4x^2}$. Prove that f is onto.

2. (5 points) What's wrong with this attempt to define $f \circ g$?

   If $f : A \to B$ and $g : B \to C$ are functions, then $f \circ g$ is the function from A to C defined by $(f \circ g)(x) = f(g(x))$.

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1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}$ by $g(x, y) = f(x) + 2f(y) - 6$. Prove that g is onto.

2. (5 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is increasing (but perhaps not strictly increasing). Dumbledore claims that f must be one-to-one. Is he correct? Briefly explain why he is or give a concrete counter-example.

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1. (10 points) Suppose that $h : \mathbb{Z} \to \mathbb{Z}$ is one-to-one. Let's define $f : \mathbb{Z}^2 \to \mathbb{Z}^2$ by $f(x, y) = (h(x) - y, 3h(x) + 1)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Recall that $\mathbb{Z}^+$ is the set of positive integers. Let's define the function $h : (\mathbb{Z}^+)^2 \to \mathbb{R}$ such that $h(d, e) = 2^d + \frac{1}{e}$. Is h onto? Briefly justify your answer.