CS 173, Fall 17 Examlet 5, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) How many different 7-letter strings can be made by selecting and rearranging letters from the word ``metalworking''? Show your work.

   Solution: The word ``metalworking'' contains 12 letters with no duplicates. We are selecting an ordered list of 7 from this set. By the permutation formula, this can be done in $\frac{12!}{5!}$ ways.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the _____ of f.

- [ ] domain
- [ ] co-domain
- [x] image
- [ ] none of these

$f : \mathbb{Z} \to \mathbb{Z}$
$f(x) = x + 4$ (x even),
$f(x) = x - 22$ (x odd)

- [x] onto
- [ ] not onto
- [ ] not a function

$g : \mathbb{Z} \to \mathbb{Z}$
$g(x) = \lfloor x \rfloor$

- [x] one-to-one
- [ ] not one-to-one
- [ ] not a function

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on exactly two mailboxes.

- [ ] true
- [x] false

$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$

- [ ] true
- [x] false

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1. (5 points) 10 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can they line up (left to right) in front of the stage without any men being next to another man?

   Solution: There are 15! ways to arrange the women.

   Then there are 16 places to put a man: between two women or at one of the ends. We have $\frac{16!}{6!}$ ways to arrange the men in these spaces.

   So the total number of different lines is is $15! \cdot \frac{16!}{6!}$.

2. (10 points) Check the (single) box that best characterizes each item.

   A function is onto if and only if its image is the same as its co-domain. — true ☑ √ — false ☐

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 3$ (x even),
   $f(x) = x - 21$ (x odd)
   — one-to-one ☑ √ — not one-to-one ☐ — not a function ☐

   $g : \mathbb{Z} \to \mathbb{R}$
   $g(x) = x + 2.137$
   — onto ☐ — not onto ☑ √ — not a function ☐

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have charm. — true ☐ — false ☑ √

   $\exists y \in \mathbb{R}^+,\ \forall x \in \mathbb{R}^+,\ xy = 1$
   ($\mathbb{R}^+$ is the positive real numbers.) — true ☐ — false ☑ √

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1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from A to $B \times C$?

   Solution: There are qn elements in $B \times C$. So there are $(qn)^p$ ways to build a function from A to $B \times C$.

2. (10 points) Check the (single) box that best characterizes each item.

A function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing if and only if it is one-to-one.	true	☐	false	☑		
$f : \mathbb{Z} \to \mathbb{Z}$ $f(x) = x + 3$ (x even), $f(x) = x - 22$ (x odd)	onto	☐	not onto	☑	not a function	☐
$g : \mathbb{R} \to \mathbb{Z}$ $g(x) = \lvert x \rvert$	one-to-one	☐	not one-to-one	☐	not a function	☑
We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there are two mailboxes with the same color.	true	☑	false	☐		
$\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ y \leq x$	true	☑	false	☐		

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1. (5 points) Suppose that $|A| = 3$ and $|B| = 3$. How many onto functions are there from A to B? Briefly justify or show work.

   Solution: Since the two sets have the same number of elements, an onto function must also be one-to-one. If two inputs mapped to the same output value, it would be impossible for the image to cover all of B. So we can use the formula for the number of permutations of 3 values: there are $3! = 6$ onto functions from A to B.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the integers is the _____ of f.

domain [√] co-domain [ ]
image [ ] none of these [ ]

$g : \mathbb{Z} \to \mathbb{Z}$
$g(x) = |x|$

one-to-one [ ] not one-to-one [√] not a function [ ]

$g : \mathbb{R} \to [0, 1]$
$g(x) = \sin(x)$

onto [ ] not onto [ ] not a function [√]

Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, there must be at least three elves with the same gift.

true [√] false [ ]

$\exists y \in \mathbb{N},\ \forall x \in \mathbb{Z},\ x^2 = y$

true [ ] false [√]

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1. (5 points) Let n and k be integers. Consider the integer powers of n from n^0 to n^k. Use the Pigeonhole Principle to show that there are two distinct (i.e. not equal) integers i and j, both between 0 and k (inclusive), such that $n^i \equiv n^j \pmod{k}$. (Your solution should be clear but does not need to be very formal.)

   Solution: For each power n^p, look at the remainder when n^p is divided by k. There are only k possible remainders, but there are $k+1$ powers in our list. So there are two powers with the same remainder, call them n^i and n^j. Since they have the same remainder, $n^i \equiv n^j \pmod{k}$.

2. (10 points) Check the (single) box that best characterizes each item.

If a function is onto, then each value in the co-domain has exactly one pre-image.	true	☐	false	☑		
$g : \mathbb{R} \to \mathbb{R}^2$ $g(x) = (x, 3x^2 + 2)$	one-to-one	☑	not one-to-one	☐	not a function	☐
$f : \mathbb{N} \to \mathbb{R}$ $f(x) = x^2 + 2$	onto	☐	not onto	☑	not a function	☐
If $f : A \to B$ is one-to-one, then	$\lvert A\rvert \geq \lvert B\rvert$	☐	$\lvert A\rvert \leq \lvert B\rvert$	☑	$\lvert A\rvert = \lvert B\rvert$	☐
$\exists t \in \mathbb{Z}^+, \ \forall p \in \mathbb{Z}^+, \ \gcd(p,t) = 1$	true	☑	false	☐		

CS 173, Fall 17 Examlet 5, Part B

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Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 14-letter strings can be made be rearranging the letters in the word ``classification''? Show your work.

   Solution: There are 14 letters total, with 2 copies of c, 3 copies of i, 2 copies of a, and 2 copies of s. So the number of possibilities is

$$\frac{14!}{3!2!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{Z}$ is a function such that $f(x) = -|x|$ then $\mathbb{N}$ is the _____ of f.
   domain ☐ co-domain ☐ image ☐ none of these ☑

   $f : \mathbb{N}^2 \to \mathbb{N}$, $f(p, q) = pq$
   one-to-one ☐ not one-to-one ☑ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = |x|$
   onto ☐ not onto ☑ not a function ☐

   We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes.
   true ☑ false ☐

   $\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = p$
   true ☑ false ☐