CS 173, Fall 17 Examlet 5, Part B

Name: _____

NetID: _____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 7-letter strings can be made by selecting and rearranging letters from the word ‘‘`metalworking`’’? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the set of all even integers is the ____ of f.

   domain ☐ co-domain ☐ image ☐ none of these ☐

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 4$ (x even),
   $f(x) = x - 22$ (x odd)

   onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$
   $g(x) = \lfloor x \rfloor$

   one-to-one ☐ not one-to-one ☐ not a function ☐

   We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on exactly two mailboxes.

   true ☐ false ☐

   $\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ y \leq x$

   true ☐ false ☐

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1. (5 points) 10 men and 15 women showed up to this week's meeting of the UIUC Swing Dance Society. How many different ways can they line up (left to right) in front of the stage without any men being next to another man?

2. (10 points) Check the (single) box that best characterizes each item.

A function is onto if and only if its image is the same as its co-domain. true ☐ false ☐

$f : \mathbb{Z} \to \mathbb{Z}$
$f(x) = x + 3$ (x even),
$f(x) = x - 21$ (x odd)
one-to-one ☐ not one-to-one ☐ not a function ☐

$g : \mathbb{Z} \to \mathbb{R}$
$g(x) = x + 2.137$
onto ☐ not onto ☐ not a function ☐

Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, the pigeonhole principle says that at least three elves have charm. true ☐ false ☐

$\exists y \in \mathbb{R}^+,\ \forall x \in \mathbb{R}^+,\ xy = 1$
($\mathbb{R}^+$ is the positive real numbers.) true ☐ false ☐

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1. (5 points) Suppose that $|A| = p$, $|B| = q$, $|C| = n$. How many different functions are there from A to $B \times C$?

2. (10 points) Check the (single) box that best characterizes each item.

   A function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing if and only if it is one-to-one. true ☐ false ☐

   $f : \mathbb{Z} \to \mathbb{Z}$
   $f(x) = x + 3$ (x even),
   $f(x) = x - 22$ (x odd) onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{R} \to \mathbb{Z}$
   $g(x) = |x|$ one-to-one ☐ not one-to-one ☐ not a function ☐

   We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there are two mailboxes with the same color. true ☐ false ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{N},\ y \leq x$ true ☐ false ☐

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1. (5 points) Suppose that $|A| = 3$ and $|B| = 3$. How many onto functions are there from A to B? Briefly justify or show work.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the integers is the _____ of f.
   domain ☐ co-domain ☐ image ☐ none of these ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = |x|$
   one-to-one ☐ not one-to-one ☐ not a function ☐

   $g : \mathbb{R} \to [0, 1]$, $g(x) = \sin(x)$
   onto ☐ not onto ☐ not a function ☐

   Each elf has exactly one gift: charm, strength, or stamina. If there are 10 elves, there must be at least three elves with the same gift.
   true ☐ false ☐

   $\exists y \in \mathbb{N},\ \forall x \in \mathbb{Z},\ x^2 = y$
   true ☐ false ☐

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1. (5 points) Let n and k be integers. Consider the integer powers of n from n^0 to n^k. Use the Pigeonhole Principle to show that there are two distinct (i.e. not equal) integers i and j, both between 0 and k (inclusive), such that $n^i \equiv n^j \pmod{k}$. (Your solution should be clear but does not need to be very formal.)

2. (10 points) Check the (single) box that best characterizes each item.

   If a function is onto, then each value in the co-domain has exactly one pre-image. true ☐ false ☐

   $g : \mathbb{R} \to \mathbb{R}^2$
   $g(x) = (x, 3x^2 + 2)$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $f : \mathbb{N} \to \mathbb{R}$
   $f(x) = x^2 + 2$ onto ☐ not onto ☐ not a function ☐

   If $f : A \to B$ is one-to-one, then $|A| \geq |B|$ ☐ $|A| \leq |B|$ ☐ $|A| = |B|$ ☐

   $\exists t \in \mathbb{Z}^+,\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = 1$ true ☐ false ☐

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1. (5 points) How many different 14-letter strings can be made be rearranging the letters in the word ‘‘`classification`’’? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{Z} \to \mathbb{Z}$ is a function such that $f(x) = -|x|$ then $\mathbb{N}$ is the ____ of f. ☐ domain ☐ co-domain ☐ image ☐ none of these

$f : \mathbb{N}^2 \to \mathbb{N}$
$f(p, q) = pq$ ☐ one-to-one ☐ not one-to-one ☐ not a function

$g : \mathbb{Z} \to \mathbb{Z}$
$g(x) = |x|$ ☐ onto ☐ not onto ☐ not a function

We painted 12 mailboxes. There were 5 colors to choose from and each mailbox is painted with a single color. By the pigeonhole principle, there is a color that appears on at least two mailboxes. ☐ true ☐ false

$\exists t \in \mathbb{N},\ \forall p \in \mathbb{Z}^+,\ \gcd(p, t) = p$ ☐ true ☐ false