CS 173, Fall 17 Examlet 7, Part A

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Working directly from the definition of divides, use (strong) induction to prove the following claim:

Claim: $2^{n+2} + 3^{2n+1}$ is divisible by 7, for all natural numbers n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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If f is a function, recall that f' is its derivative. Recall the product rule: if $f(x) = g(x)h(x)$, then $f'(x) = g'(x)h(x) + g(x)h'(x)$. Assume we know that the derivative of $f(x) = x$ is $f'(x) = 1$.

Use (strong) induction to prove the following claim:

For any positive integer n, if $f(x) = x^n$ then $f'(x) = nx^{n-1}$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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Use (strong) induction to prove the following claim:

For any natural number n, $\sum_{p=0}^{n} 3(-1/2)^p = 2 + (-1/2)^n$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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Use (strong) induction to prove the following claim:

For all natural numbers n, $\sum_{p=0}^{n}(2p+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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Let's say that a set of polygonal regions in the plane is "properly colored" if regions sharing an edge never have the same color.

Suppose that we draw n lines in the plane, in general position (no lines are parallel, no point belongs to more than two lines). The lines divide up the plane into a set of regions. Use (strong) induction to prove that, for any positive integer n, this set of regions can be properly colored with two colors.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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Use (strong) induction to prove the following claim:

Claim: For all integers a, b, n, $n \geq 1$, if $a \equiv b \pmod 7$ then $a^n \equiv b^n \pmod 7$.

Use this definition in your proof: $x \equiv y \pmod p$ if and only if $x = y + kp$ for some integer k.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: