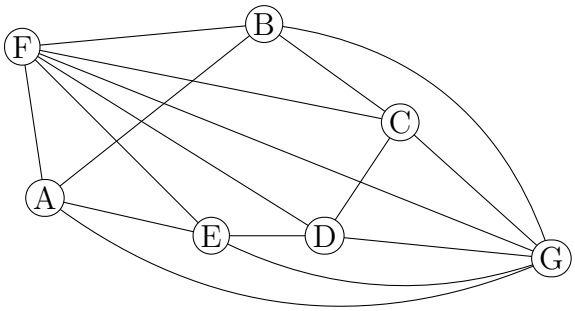
CS 173, Fall 17 Examlet 7, Part B

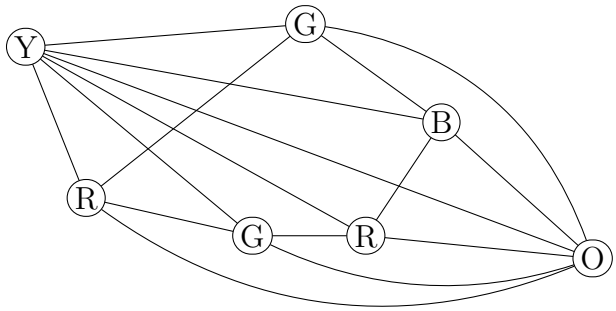
Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.





Solution: The chromatic number is five. The picture above shows how to color it with five colors (upper bound).

For the lower bound, the graph contains a W_5 whose hub is F and whose rim contains nodes A, B, C, D, E. Coloring a W_5 requires four colors. Then the node G is connected to all six nodes in the W_5, so it needs a different, fifth color.

2. (6 points) Check the (single) box that best characterizes each item.

Chromatic number of a graph with maximum vertex degree D

$= D$	☐	$= D+1$	☐
$\geq D+1$	☐	$\leq D+1$	☑

We have 30 tablespoons of filling and each bun will require one tablespoon. 30 is ____ on how many buns we can make.

an upper bound on	☑	exactly	☐
a lower bound on	☐	not a bound on	☐

$\sum_{k=0}^{n+1} 2^k$

$2^{n+1}+1$	☐	$2^{n+2}-1$	☑	$2^{n+2}-2$	☐	$2^{n+1}-1$	☐

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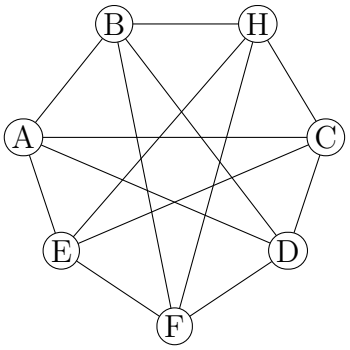
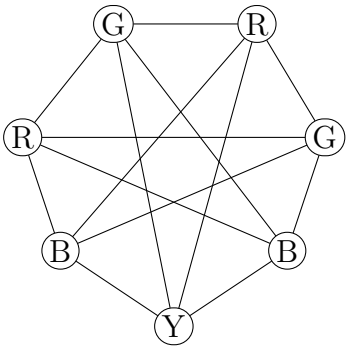
1. (9 points) What is the chromatic number of the graph below? Justify your answer.





Solution: The chromatic number is 4. The picture above shows that the graph can be colored with four colors (upper bound).

To show the lower bound, let's try to color the graph with three colors. First color the triangle ABD as shown in the above picture. Then C must be colored G and E must be colored B. The colorings on C and E imply that H must be colored R.

But none of the three colors is possible for F. So three colors isn't enough, i.e. we have a lower bound of 4.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{i=1}^{p-1} \frac{i}{p}$ $\frac{p(p-1)}{2}$ ☐ $\frac{p(p+1)}{2}$ ☐ $\frac{(p+1)}{2}$ ☐ $\frac{(p-1)}{2}$ ☑

10 people rowed across Lake Tahoe in my canoe. 10 is ____ how many people the canoe can carry.

an upper bound on ☐ exactly ☐

a lower bound on ☑ not a bound on ☐

Chromatic number of a graph containing a W_7. ≥ 3 ☐ ≥ 4 ☑ ≥ 7 ☐ can't tell ☐

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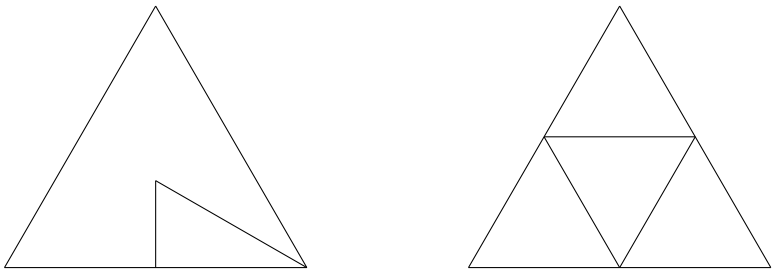
1. (9 points) Tomas wants to plant his tomatoes so that plants are more than 1 foot apart. His garden bed is an an equilateral triangle with each side 2 feet long. Prove that four is the maximum number of tomatoes he can plant.

   Solution: We need to show that four is possible (lower bound) and that five is not possible (upper bound).

   Lower bound: Put one tomato at each corner of the bed and one tomato in the exact center. Plants at two corners are 2 feet apart. You can see from the lefthand figure below that the plant in the center is more than a foot from each corner.



1

   Upper bound: Divide the bed into four small triangles with side length 1, as shown above right. Two points in the same small triangle are ≤ 1 foot apart, so we can't put two tomatoes in the same small triangle. So we can't plant more than four tomatoes.

2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{i=1}^{p-1} i$ $\frac{(p-1)^2}{2}$ ☐ $\frac{(p-1)(p+1)}{2}$ ☐ $\frac{p(p+1)}{2}$ ☐ $\frac{p(p-1)}{2}$ ☑

Putting 10 people in the canoe caused it to sink. 10 is ____ how many people the canoe can carry.

an upper bound on ☑ exactly ☐

a lower bound on ☐ not a bound on ☐

Chromatic number of a connected graph with 10 nodes. ≤ 2 ☐ $= 2$ ☐ ≥ 2 ☑ can't tell ☐

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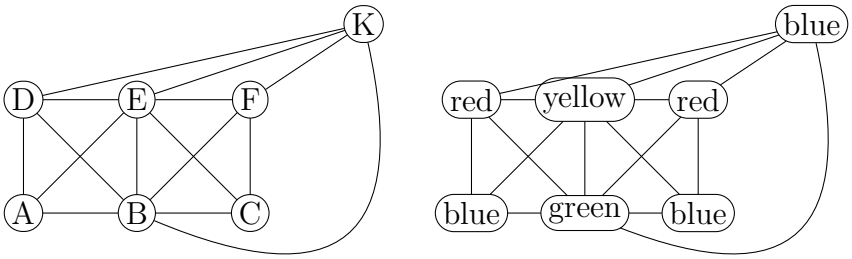
Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) What is the chromatic number of the graph below? Justify your answer.



Solution: The chromatic number is 4. The righthand picture shows that four colors are sufficient (upper bound).

To show that four colors are required (lower bound), notice that A, B, C, and E form a K_4.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{k=0}^{n-1} 2^k$ $2^n - 2$ ☐ $2^n - 1$ ☑ $2^{n-1} - 1$ ☐ $2^{n+1} - 1$ ☐

C_5 is a subgraph of graph H. 3 is ____ the chromatic number of H.

an upper bound on ☐ exactly ☐
a lower bound on ☑ not a bound on ☐

Exactly 40 books fit in my suitcase by volume, but I haven't checked their total weight. 40 is ____ how many books the suitcase can hold.

an upper bound on ☑ exactly ☐
a lower bound on ☐ not a bound on ☐

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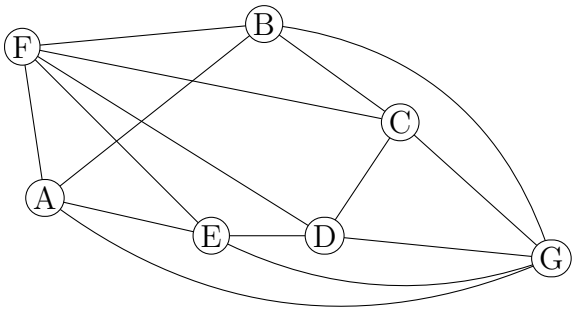
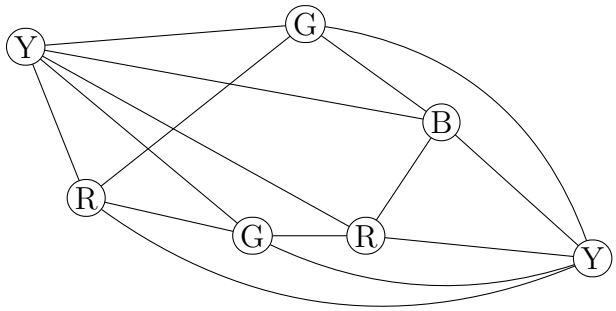
1. (9 points) What is the chromatic number of the graph below? Justify your answer.





Solution: The chromatic number is four. The picture above shows how to color it with four colors (upper bound).

For the lower bound, the graph contains a W_5 whose hub is F and whose rim contains nodes A, B, C, D, E. Coloring a W_5 requires four colors.

2. (6 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n} 2^k$ $2^{n+1}-1$ ☐ $2^{n+1}-2$ ☑ $2^{n+1}-3$ ☐ 2^n-1 ☐

C_5 is a subgraph of graph H. 5 is ____ the chromatic number of H.

an upper bound on ☐ exactly ☐

a lower bound on ☐ not a bound on ☑

Chromatic number of a bipartite graph with at least one edge

1 ☐ 2 ☑ 3 ☐ can't tell ☐

Name:_____

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1. (11 points) Let's define two sets as follows:

$$A = \{(4 - t^2, t + 1) : t \in \mathbb{R}\}$$

$$B = \{(x, y) \in \mathbb{R}^2 : x = 3 + 2y - y^2\}$$

Prove that $A = B$ by proving two subset inclusions.

Solution:

A $\subseteq$ **B**: Let $(x, y) \in A$. Then $(x, y) = (4 - t^2, t + 1)$ for some real number t. So $x = 4 - t^2$ and $y = t + 1$. Then $t = y - 1$. So $x = 4 - t^2 = 4 - (y - 1)^2 = 4 - (y^2 - 2y + 1) = 3 + 2y - y^2$. So $(x, y) \in B$.

B $\subseteq$ **A**: Let $(x, y) \in B$. Then $x = 3 + 2y - y^2$. Let $t = y - 1$. Then $y = t + 1$. Furthermore $x = 4 - (1 - 2y + y^2) = 4 - (y - 1)^2 = 4 - t^2$. So $(x, y) = (4 - t^2, t + 1)$, where t is a real number. And therefore $(x, y) \in A$.

Since $A \subseteq B$ and $B \subseteq A$, $A = B$, which is what we needed to show.

2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{k=3}^{n} k^7$ $\sum_{p=1}^{n-2} p^9$ ☐ $\sum_{p=1}^{n-2} k^7$ ☐ $\sum_{p=1}^{n-2} k^9$ ☐ $\sum_{p=1}^{n-2} (p+2)^7$ ☑

Chromatic number of $K_{m,n}$. 2 ☑ 3 ☐ 4 ☐ can't tell ☐