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(20 points) Use (strong) induction to prove that $a - b$ divides $a^n - b^n$, for any integers a and b and any natural number n.

Hint: $(a^n - b^n)(a + b) = (a^{n+1} - b^{n+1}) + ab(a^{n-1} - b^{n-1})$, for any real numbers a and b.

Let a and b be integers.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $f : \mathbb{N}^2 \to \mathbb{N}$ is defined by

$f(n, 0) = f(n, n) = 1$, for any natural number n

$f(n, a) = f(n-1, a-1) + f(n-1, a)$, for all n and a such that $1 \leq a \leq n-1$

Use (strong) induction to prove that $f(n, a) = \frac{n!}{a!(n-a)!}$ for any natural numbers a and n, where $n \geq a$. Hint: use n as your induction variable. At each step, make sure the equations work for an arbitrary natural number $a \leq n$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: [First deal with two special cases: $f(k, 0)$ and $f(k, k)$.]

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(20 points) (20 points) Suppose that $f : \mathbb{N} \to \mathbb{Z}$ is defined by

$f(0) = 2$ $f(1) = 5$ $f(2) = 15$

$f(n) = 6f(n-1) - 11f(n-2) + 6f(n-3)$, for all $n \geq 3$

Use (strong) induction to prove that $f(n) = 1 - 2^n + 2 \cdot 3^n$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is defined by

$f(0) = 0$ $\quad f(1) = 1$

$f(n) = f(n-1) + f(n-2)$ for all $n \geq 2$.

Let $a = \frac{1+\sqrt{5}}{2}$ and $b = \frac{1-\sqrt{5}}{2}$. Use (strong) induction to prove that $f(n) = \dfrac{a^n - b^n}{a - b}$.

First show that $a^2 = a + 1$ **and** $b^2 = b + 1$**:**

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $g : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by

$g(1) = 1$

$g(2) = 8$

$g(n) = g(n-1) + 2g(n-2)$

Use (strong) induction to prove that $g(n) = 3 \cdot 2^{n-1} + 2(-1)^n$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove that

$$\prod_{p=1}^{n} \frac{m+1-p}{p} = \frac{m!}{n!(m-n)!}$$

for any positive integers m and n where $m \geq n$. Hint: use n as your induction variable. At each step, make sure the the equations work for an arbitrary integer $m \geq n$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: