CS 173, Fall 17 Examlet 9, Part B

Name:_____

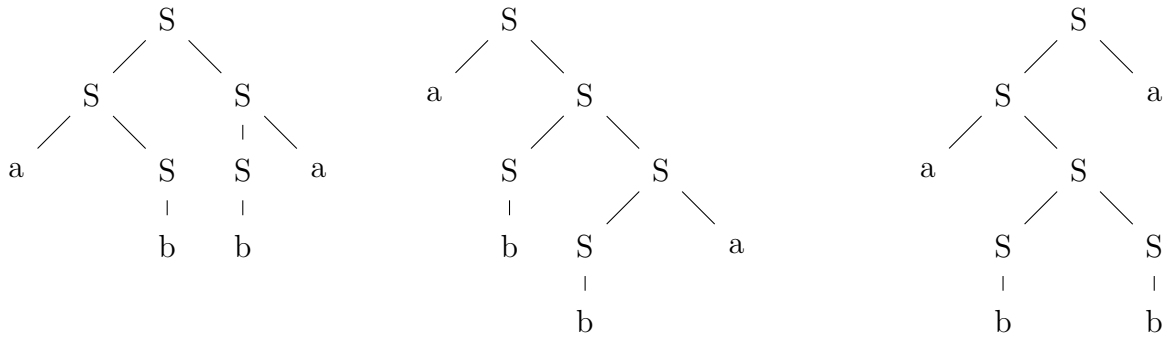
NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string `abba` that match this grammar.

$$S \rightarrow S\,S \mid a\,S \mid S\,a \mid b$$

Solution:



2. (4 points) Check the (single) box that best characterizes each item.

The mathematical symbol for an empty (zero-length) string

$\emptyset$ ☐ e ☐ ϵ ☑ NULL ☐

Number of nodes at level k in a full complete binary tree.

2^k ☑ $2^k - 1$ ☐ $2^{k+1} - 1$ ☐ 2^{k-1} ☐

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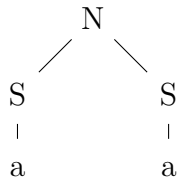
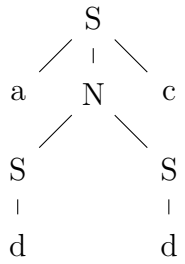
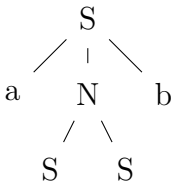
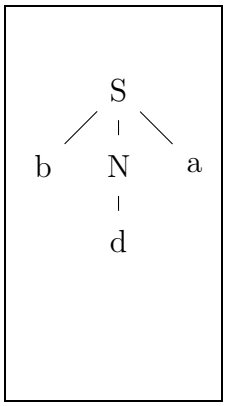
1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$S \rightarrow b\ N\ a \mid a\ N\ c \mid a$$
$$N \rightarrow S\ S \mid d$$









2. (4 points) Check the (single) box that best characterizes each item.

Number of bit strings of length k. 2^k [√] $2^k - 1$ [ ] 2^{k-1} [ ] k [ ]

A full m-ary tree with i internal nodes has $mi + 1$ nodes total. always [√] sometimes [ ] never [ ]

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1. (8 points) Consider the following grammar G

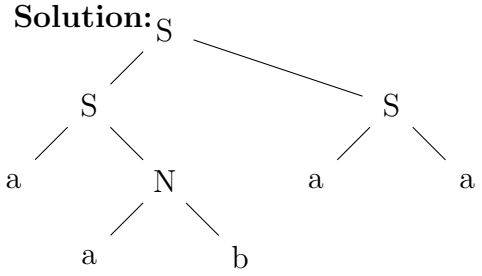
$$\begin{aligned} S &\rightarrow S\ S \mid a\ N \mid a\ a \\ N &\rightarrow S\ a \mid a\ b \end{aligned}$$

S is the only start symbol. The terminal symbols are a and b

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

`aabaa`

Solution:



`ab`

Solution: This is impossible. An ab sequence must come from the rule $N \rightarrow a\ b$. But N isn't a start symbol and getting to this rule from S would require adding something else to the string.

2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at least $2^{h+1} - 1$ nodes. true ☐ false ☑

The level of the root node in a tree of height h. 0 ☑ 1 ☐ $h-1$ ☐ h ☐ $h+1$ ☐

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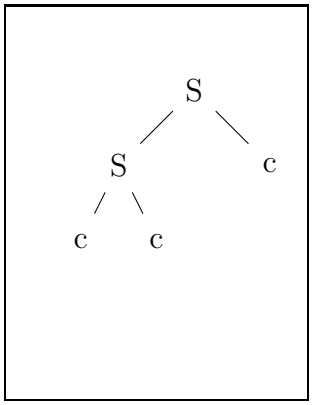
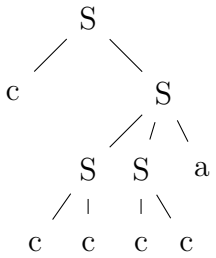
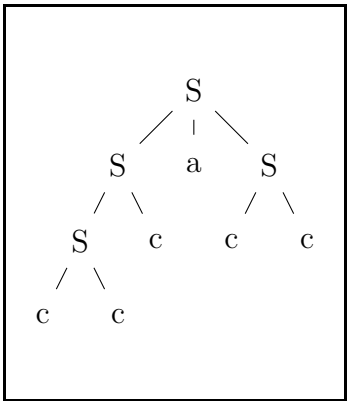
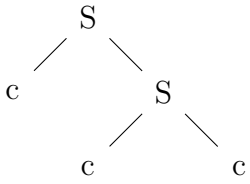
1. (8 points) Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \rightarrow S\ a\ S \mid S\ c \mid c\ c$$









2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at least $2^h - 1$ nodes. true ☐ false ☑

Number of bit strings of length $\leq k$. 2^k ☐ $2^k - 1$ ☐ 2^{k-1} ☐ $2^{k+1} - 1$ ☑

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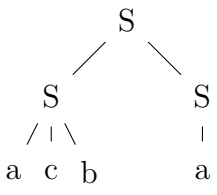
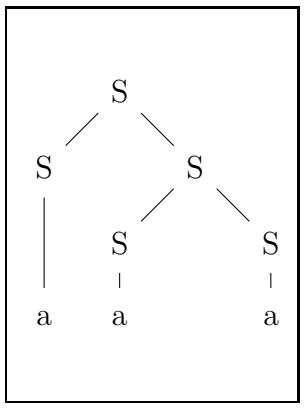
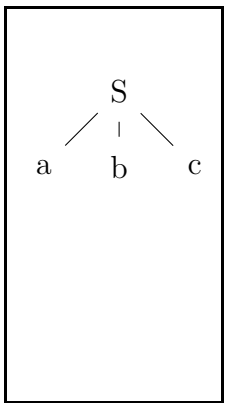
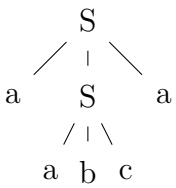
1. (8 points) Here is a grammar with start symbol S and terminals symbols a,b, and c. Circle the trees that match the grammar.

$$S \rightarrow S\ S \mid a\ b\ c \mid a$$









2. (4 points) Check the (single) box that best characterizes each item.

The number of nodes in a binary tree of height h

$\geq 2^h$	☐	$2^{h+1} - 1$	☐
$\leq 2^{h+1} - 1$	☑	$\geq 2^{h+1} - 1$	☐

The diameter of a tree of height h.

$\leq h$	☐	h	☐	$h + 1$	☐
$2h$	☐	$\leq 2h$	☑		

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1. (8 points) Consider the following grammar G

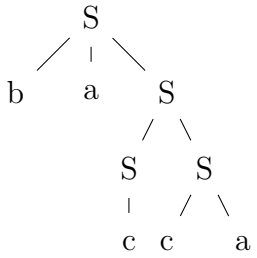
   $S \to b\ a\ S \mid S\ S \mid c \mid c\ a$

   S is the only start symbol. The terminal symbol are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   `baba`
   Solution: This is impossible. The only rule generating `ba` is $S \to b\ a\ S$. So `baba` requires two applications of this rule. But that will leave us with an extra S at the end and S isn't a terminal.

   `bacca`
   Solution:



2. (4 points) Check the (single) box that best characterizes each item.

   The number of leaves in a binary tree of height h

   2^h ☐ $2^{h+1}-1$ ☐ $\geq 2^h$ ☐ $\leq 2^h$ ☑

   The diameter of a full, complete 7-ary tree of height h.

   $\leq h$ ☐ h ☐ $h+1$ ☐

   $2h$ ☑ $7h$ ☐ $7h+1$ ☐