Name:_____

NetID:_____   **Lecture:**   A   B

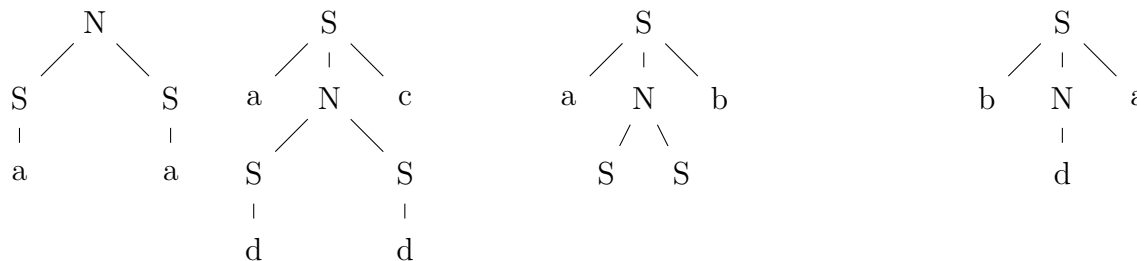**Discussion:**   **Thursday**   **Friday**   9   10   11   12   1   2   3   4   5   6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string `abba` that match this grammar.

$$S \quad \rightarrow \quad S\ S \mid a\ S \mid S\ a \mid b$$

2. (4 points) Check the (single) box that best characterizes each item.

The mathematical symbol for an empty (zero-length) string $\emptyset$ ☐   e ☐ ϵ ☐   NULL ☐

Number of nodes at level k in a full complete binary tree. 2^k ☐ $2^k - 1$ ☐ $2^{k+1} - 1$ ☐ 2^{k-1} ☐

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1. (8 points)   Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$S \rightarrow b\ N\ a \mid a\ N\ c \mid a$$
$$N \rightarrow S\ S \mid d$$

2. (4 points) Check the (single) box that best characterizes each item.

Number of bit strings of length k. 2^k ☐ $2^k - 1$ ☐ 2^{k-1} ☐ k ☐

A full m-ary tree with i internal nodes has $mi + 1$ nodes total.   always ☐   sometimes ☐   never ☐

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Discussion: **Thursday**   **Friday**   9   10   11   12   1   2   3   4   5   6

1. (8 points) Consider the following grammar G

$$
\begin{aligned}
S &\rightarrow S\,S \mid a\,N \mid a\,a \\
N &\rightarrow S\,a \mid a\,b
\end{aligned}
$$

S is the only start symbol. The terminal symbols are a and b

Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

aabaa   ab

2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at least $2^{h+1} - 1$ nodes.   true ☐   false ☐

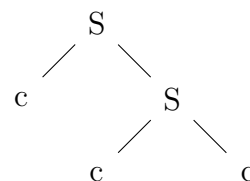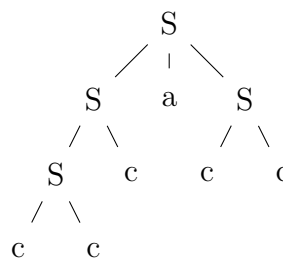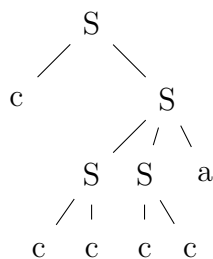The level of the root node in a tree of height h.   0 ☐   1 ☐ $h-1$ ☐ h ☐ $h+1$ ☐

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1. (8 points)   Here is a grammar, with start variable S and terminals a and c. Circle the trees that match the grammar.

$$S \;\rightarrow\; S\,a\,S \;\mid\; S\,c \;\mid\; c\,c$$

Tree 1:
```
      S
    /   \
   S     c
  / \
 c   c
```

Tree 2:
```
    S
  /   \
 c     S
      /|\
     S S  a
    /| |\
   c c c  c
```

Tree 3:
```
      S
    / | \
   S  a  S
  / \   / \
 S   c c   c
/ \
c  c
```

Tree 4:
```
    S
  /   \
 c     S
      / \
     c   c
```

2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at least $2^h - 1$ nodes.   true ☐   false ☐

Number of bit strings of length $\leq k$. 2^k ☐ $2^k - 1$ ☐ 2^{k-1} ☐ $2^{k+1} - 1$ ☐

Name:_____

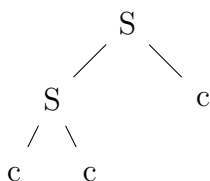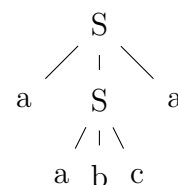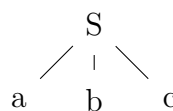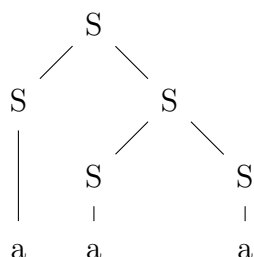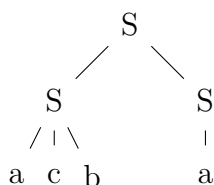NetID:_____   Lecture: A   B

Discussion: Thursday   Friday   9   10   11   12   1   2   3   4   5   6

1. (8 points)   Here is a grammar with start symbol S and terminals symbols a,b, and c. Circle the trees that match the grammar.

$$S \quad \rightarrow \quad S\ S \mid a\ b\ c \mid a$$

2. (4 points) Check the (single) box that best characterizes each item.

The number of nodes in a binary tree of height h

$\geq 2^h$	☐	$2^{h+1} - 1$	☐
$\leq 2^{h+1} - 1$	☐	$\geq 2^{h+1} - 1$	☐

The diameter of a tree of height h.

$\leq h$	☐	h	☐	$h+1$	☐
$2h$	☐	$\leq 2h$	☐		

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1. (8 points)   Consider the following grammar G

   $$S \rightarrow b\,a\,S \mid S\,S \mid c \mid c\,a$$

   S is the only start symbol. The terminal symbol are a, b, and c.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

        `baba`   `bacca`

2. (4 points) Check the (single) box that best characterizes each item.

   The number of leaves in a binary tree of height h 2^h ☐ $2^{h+1} - 1$ ☐ $\geq 2^h$ ☐ $\leq 2^h$ ☐

   The diameter of a full, complete 7-ary tree of height h. $\leq h$ ☐ h ☐ $h+1$ ☐

                                             $2h$ ☐ $7h$ ☐ $7h+1$ ☐