CS 173, Fall 17 Examlet 10, Part A

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: For any positive integer n and any positive reals $a_1, \ldots, a_n$,

$$\prod_{p=1}^{n}(1+a_p) \geq 1 + \sum_{p=1}^{n} a_p$$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: For all integers $n \geq 2$, $(2n)! > 2^n n!$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) Use (strong) induction to prove the following claim:

Claim: For any sets A_1, A_2, ..., A_n, $|A_1 \cup A_2 \cup \ldots \cup A_n| \leq |A_1| + |A_2| + \ldots + |A_n|$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: remember the "Inclusion-Exclusion" formula for computing $|A \cup B|$ in terms of $|A|$, $|B|$, $|A \cap B|$.

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(15 points) Use (strong) induction to prove the following claim. You may use the fact that $\sqrt{2} \leq 1.5$.

Claim: For any positive integer n, $\sum_{p=1}^{n} \frac{1}{\sqrt{p}} \geq 2\sqrt{n+1} - 2$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: Hint: notice that $(\sqrt{x+1} - \sqrt{x+2}\ )^2 \geq 0$. What does this imply about $2\sqrt{x+1}\sqrt{x+2}$?

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(15 points) Let function $f : \mathbb{Z}^+ \to \mathbb{N}$ be defined by

$f(1) = 0$

$f(n) = 1 + f(\lfloor n/2 \rfloor)$, for $n \geq 2$,

Use (strong) induction on n to prove that $f(n) \leq \log_2 n$ for any positive integer n. You cannot assume that n is a power of 2. However, you can assume that the log function is increasing (if $x \leq y$ then $\log x \leq \log y$) and that $\lfloor x \rfloor \leq x$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(15 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Use (strong) induction to prove the following claim:

Claim: For any positive integer n and any positive reals $a_1, \ldots, a_n$,

$$\prod_{p=1}^{n}(1 - a_p) \geq 1 - \sum_{p=1}^{n} a_p$$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: