CS 173, Fall 17 Examlet 10, Part B

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 9 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 3.

   $T(3) = 7$ $T(n) = 4T\left(\frac{n}{3}\right) + 5n$

   (a) The height:

   (b) Value in each node at level k:

   (c) Sum of the work in all the leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $(3^n)^2$ 10 $0.001n^3$ $30 \log n$ $n \log(n^7)$ $8n! + 18$ $3n^2$

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $O(h(x))$ and $g(x) \ll f(x))$. Must $f(x) + g(x)$ be $O(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $T(n) = 3T(n/3) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = c$ $T(n) = 3T(n/3) + n$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$n!$ $O(2^n)$ ☐ $\Theta(2^n)$ ☐ neither of these ☐

$n^{log_2 4}$ grows faster than n^2 ☐ slower than n^2 ☐ at the same rate as n^2 ☐

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $\Theta(h(x))$, $g(x)$ is $\Theta(h(x))$, and $f(x) > g(x)$ for any input x. Must $f(x) - g(x)$ be $\Theta(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$
$T(n) = 2T(n/2) + n$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$T(1) = d$
$T(n) = 2T(n-1) + c$

$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

$n^{log_2 3}$ grows faster than n ☐ slower than n ☐ at the same rate as n ☐

Suppose $f(n)$ is $\Theta(g(n))$.
Will $g(n)$ be $\Theta(f(n))$? no ☐ perhaps ☐ yes ☐

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1. (7 points) You found the following claim on a hallway whiteboard. Suppose that f and g are increasing functions from the reals to the reals, for which all output values are > 1. If $f(x)$ is $O(g(x))$, then $\log(f(x))$ is $O(\log(g(x)))$. Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

3^n is $\Theta(2^n)$ ☐ $O(2^n)$ ☐ neither of these ☐

Dividing a problem of size n into m subproblems, each of size n/k, has the best big-Θ running time when $k < m$ ☐ $k = m$ ☐ $k > m$ ☐ $km = 1$ ☐

$T(1) = d$
$T(n) = T(n/2) + n$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = T(n-1) + n$

$\Theta(\log n)$ ☐ $\Theta(\sqrt{n})$ ☐ $\Theta(n)$ ☐ $\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐ $\Theta(n^3)$ ☐ $\Theta(2^n)$ ☐ $\Theta(3^n)$ ☐

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(4) = 7$ $T(n) = 5T\left(\frac{n}{2}\right) + n$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $30\log(n^{17})$ $\sqrt{n} + n! + 18$ $\frac{n \log n}{7}$ $(10^{10^{10}})n$ $0.001n^3$ 2^n $8n^2$

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1. (7 points) Suppose that f and g are functions from the reals to the reals, such that f is $\Theta(g)$. Must g be $O(f)$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$ $T(n) = 2T(n/2) + n^2$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n-1) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$n^{log_4 2}$ grows faster than n^2 ☐ slower than n^2 ☐ at the same rate as n^2 ☐

$\log_5 n$ is $\Theta(\log_3 n)$ ☐ $O(\log_3 n)$ ☐ neither of these ☐