CS 173, Fall 17 Examlet 12, Part A

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(8 points) Marsha has 25 cups, identical except that 10 are red, 8 are blue, and 7 are green. How many ways can she make an (ordered) sequence of 25 cups?

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

There is a dorm room d, such that d has green walls and d has no window.

(2 points) Check the (single) box that best characterizes each item.

Suppose you want to take 5 classes next semester, out of 25 classes being offered. You must take STATS 100. How many different choices do you have?

$\frac{24!}{20!}$ ☐ $\frac{24!}{20!4!}$ ☐

$\frac{(24+3)!}{24!3!}$ ☐ $\frac{25!}{20!5!}$ ☐

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(9 points) Use proof by contradiction to show that, for any graph G and any two nodes a and b, the shortest walk from a to b does not contain any repeated nodes.

(6 points) Suppose a set S has 11 elements. How many subsets of S have an even number of elements? Express your answer as a summation. Briefly justify or show work.

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(8 points) Suppose we know (e.g. from the binomial theorem) that $\sum_{k=0}^{n} \binom{n}{k} = 2^n$. Use this to show that $\sum_{k=1}^{n} k \cdot \binom{n}{k} = n \cdot 2^{n-1}$. Show your work.

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every tree t, if t grows in Canada, then t is not tall or t is a conifer.

(2 points) Check the (single) box that best characterizes each item.

How many ways can I choose a set of 5 bagels, if there are 10 types of bagels and all 5 bagels must be different types?

$\frac{10!}{5!5!}$ ☐	$\frac{14!}{10!4!}$ ☐	$\frac{14!}{9!5!}$ ☐
$\frac{15!}{10!5!}$ ☐	10^5 ☐	5^{10} ☐

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(9 points) Researchers recorded phone calls between pairs of two (different) people, never repeating the same pair of people. The calls used n people ($n \geq 2$), but it's possible some people were not in any conversation. Use proof by contradiction to show that two people were in the same number of conversations.

(6 points) Use the binomial theorem to find a closed form for the summation $\sum_{k=0}^{n} \binom{n}{k}$. Make sure it's clear how you used the theorem.

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(9 points) Use proof by contradiction to show that, for any integer n, at least one of the three integers n, $2n+1$, $4n+3$ is not divisible by 7.

(6 points) Suppose a set S has 11 elements. How many subsets of S have five or fewer elements? Express your answer as a summation. Briefly justify or show work.

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(8 points) Recall that nodes in a full binary tree have either zero or two children. Suppose we are building a full binary tree with unlabelled nodes whose leaves are all at levels k or $k+1$, with p leaves at level $k+1$. How many different ways can we construct such a tree? Briefly justify your answer.

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

If it is raining, then there is a cyclist c such that c is getting wet.

(2 points) Check the (single) box that best characterizes each item.

The number of ways to select a set of 17 flowers chosen from 4 possible varieties (one or more of each variety).

$\binom{16}{3}$ ☐ $\binom{16}{4}$ ☐ $\binom{20}{3}$ ☐

$\binom{20}{4}$ ☐ $\binom{21}{3}$ ☐ 4^{17} ☐