CS 173, Fall 17 Examlet 12, Part B

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **9** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(7 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. Suppose that $m = ab$, where a and b are two different primes. Express $f(m)$ in terms of $f(a)$ and $f(b)$. Briefly justify your answer.

Solution: $f(m)$ contains all multiples of m. Since $m = ab$, and a and b are distinct primes, this means that $f(m)$ contains all numbers that are multiples of both a and b. In other words, these are numbers that are in both $f(a)$ and $f(b)$. So $f(m) = f(p) \cap f(q)$.

(8 points) Check the (single) box that best characterizes each item.

$|\{A \subseteq \mathbb{Z}_4 \ : \ |A| \text{ is even}\}|$ 1 ☐ 6 ☐ 7 ☐ 8 ☑ infinite ☐

$\binom{n}{1}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☑ undefined ☐

There is a set A such that $|\mathbb{P}(A)| \leq 2$. true ☑ false ☐

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is an integer ☐ a set of integers ☑ undefined ☐ one or more integers ☐ a power set ☐

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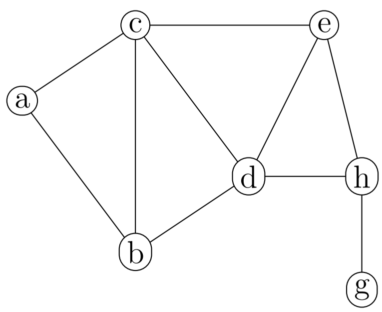
Graph G with set of nodes V is shown below. Recall that $deg(n)$ is the degree of node n. Let's define $f : \mathbb{N} \to \mathbb{P}(V)$ by $f(k) = \{n \in V : \deg(n) = k\}$. Also let $T = \{f(k) \mid k \in \mathbb{N}\}$.



(6 points) Fill in the following values:

$f(4) =$
Solution: $\{c, d\}$
$f(1) =$
Solution: $\{g\}$
$|T| =$
Solution: 5. (The distinct members are f(0), f(1), f(2), f(3), and f(4).)

(7 points) Is T a partition of V? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: No, it is not a partition. There is no partial overlap between the sets in T and they cover all nodes in V. However, T contains the empty set (e.g. as the value of $f(17)$).

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$	always ☑	sometimes ☐	never ☐

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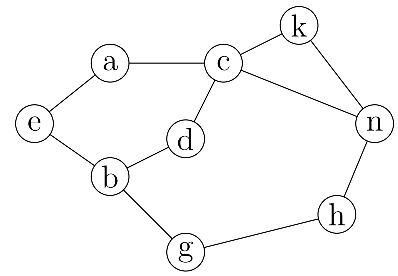
Graph G is shown at right with set of nodes V and set of edges E. Let $M : (V, \mathbb{N}) \to \mathbb{P}(V)$ be defined by
$M(x, n) = \{y \in V \mid \text{ there is a path of length } n \text{ from } x \text{ to } y\}$.
Let $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$.



(6 points) Give the value of $M(c, n)$, for all values of n from 0 to 3.

Solution: $M(c, 0) = \{c\}$ $M(c, 1) = \{a, d, n, k\}$ $M(c, 2) = \{b, e, n, h, k\}$
$M(c, 3) = \{b, e, g, h\}$

(7 points) Is $P(c)$ a partition of V? For each of the three conditions required to be a partition, explain why $P(c)$ does or doesn't satisfy that condition.

Solution: $P(c)$ is not a partition of V. $P(c)$ does cover all of V. However, some of its elements have partial overlap, e.g. $M(c, 2)$ and $M(c, 3)$. Also, since there are only 9 nodes in the graph, no path has length greater than 8. So $M(c, 9) = \emptyset$ and therefore $P(c)$ contains the empty set.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ always ☐ sometimes ☐ never ☑ ($\surd$)

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(7 points) Let $f : \mathbb{Z}^+ \to \mathbb{P}(\mathbb{Z}^+)$ be defined by $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$. For which positive integers a and b is $f(a)$ a subset of $f(b)$? Briefly justify your answer.

Solution: $f(a) \subseteq f(b)$ is true if and only if every multiple of a is also be a multiple of b. This occurs exactly when a is a multiple of b.

(8 points) Check the (single) box that best characterizes each item.

If $f : \mathbb{N} \to \mathbb{P}(\mathbb{Q})$ then $f(3)$ is

a rational ☐ a set of rationals ☑ undefined ☐

one or more rationals ☐ a power set ☐

$\{\{a, b\}, c\} = \{a, b, c\}$

true ☐ false ☑

Set B is a partition of a finite set A. Then

$|B| \leq 2^{|A|}$ ☐ $|B| \leq |A|$ ☑

$|B| = 2^{|A|}$ ☐ $|B| \leq |A + 1|$ ☐

$\binom{n}{1}$

-1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☑ undefined ☐

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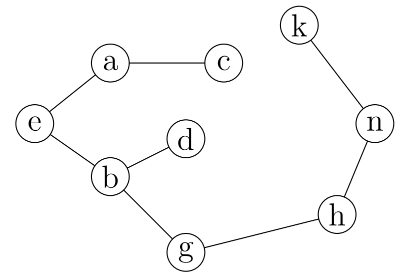
Graph G is shown at right with set of nodes V and set of edges E. Let $M : (V, \mathbb{N}) \rightarrow \mathbb{P}(V)$ be defined by $M(x, n) = \{y \in V \mid \text{ there is a path of length } n \text{ from } x \text{ to } y\}$. Let $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$.



(6 points) Give the value of $M(g, n)$, for all values of n from 0 to 3.

Solution: $M(g, 0) = \{g\}$ $M(g, 1) = \{b, h\}$ $M(g, 2) = \{e, d, n\}$ $M(g, 3) = \{a, k\}$

(7 points) Is $P(g)$ a partition of V? For each of the three conditions required to be a partition, explain why $P(g)$ does or doesn't satisfy that condition.

Solution: $P(g)$ is not a partition of V. $P(g)$ does cover all of V. Because the graph has no cycles, each node is in exactly one of the subsets in $P(g)$, so no partial overlap. However, no path has length greater than 4. So $M(g, 5) = \emptyset$ and therefore $P(g)$ contains the empty set.

(2 points) Check the (single) box that best characterizes each item.

$\binom{n}{n}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ n ☐ undefined ☐

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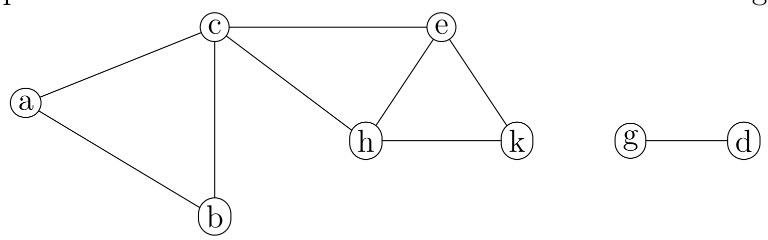
Graph G is shown below with set of nodes V and set of edges E.



Let $F : V \to \mathbb{P}(V)$ such that $F(n) = \{v \in V \mid \text{ there is a cycle containing } n \text{ and } v\}$.
Let $T = \{F(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$F(g) =$

Solution: $\emptyset$

$F(b) =$

Solution: $\{a, b, c\}$

$F(k) =$

Solution: $\{c, e, k, h\}$

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

Solution: No, it is not a partition of V. There is partial overlap between $F(b)$ and $F(k)$. T contains the empty set because $f(g) = \emptyset$. And some vertices (e.g. g) do not belong to any cycles and therefore aren't in any elements of T.

(2 points) Check the (single) box that best characterizes each item.

$\binom{n}{0}$ -1 ☐ 0 ☐ 1 ☑ 2 ☐ n ☐ undefined ☐