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Lecture:    A    B

Discussion:    Thursday    Friday    9    10    11    12    1    2    3    4    5    6

(7 points) Let  $f : \mathbb{Z}^+ \rightarrow \mathbb{P}(\mathbb{Z}^+)$  be defined by  $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$ . Suppose that  $m = ab$ , where  $a$  and  $b$  are two different primes. Express  $f(m)$  in terms of  $f(a)$  and  $f(b)$ . Briefly justify your answer.

(8 points) Check the (single) box that best characterizes each item.

$|\{A \subseteq \mathbb{Z}_4 : |A| \text{ is even}\}|$     1 ☐    6 ☐    7 ☐    8 ☐    infinite ☐

$\binom{n}{1}$     -1 ☐    0 ☐    1 ☐    2 ☐    n ☐    undefined ☐

There is a set  $A$  such that  
 $|\mathbb{P}(A)| \leq 2$ .

true ☐    false ☐

If  $f : \mathbb{R} \rightarrow \mathbb{P}(\mathbb{Z})$   
 then  $f(17)$  is

an integer ☐    a set of integers ☐    undefined ☐  
 one or more integers ☐    a power set ☐

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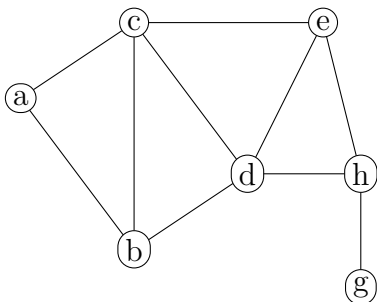
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Graph  $G$  with set of nodes  $V$  is shown below. Recall that  $\deg(n)$  is the degree of node  $n$ . Let's define  $f : \mathbb{N} \rightarrow \mathbb{P}(V)$  by  $f(k) = \{n \in V : \deg(n) = k\}$ . Also let  $T = \{f(k) \mid k \in \mathbb{N}\}$ .

(6 points) Fill in the following values:

 $f(4) =$  $f(1) =$  $|T| =$ 

(7 points) Is  $T$  a partition of  $V$ ? For each of the conditions required to be a partition, briefly explain why  $T$  does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$$

always

☐

sometimes

☐

never

☐

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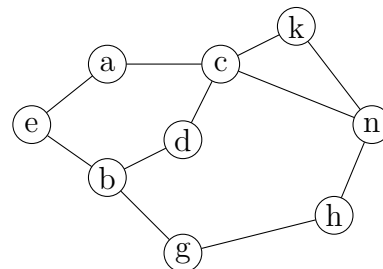
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Graph  $G$  is shown at right with set of nodes  $V$  and set of edges

$E$ . Let  $M : (V, \mathbb{N}) \rightarrow \mathbb{P}(V)$  be defined by

$M(x, n) = \{y \in V \mid \text{there is a path of length } n \text{ from } x \text{ to } y\}$ .

Let  $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$ .



(6 points) Give the value of  $M(c, n)$ , for all values of  $n$  from 0 to 3.

(7 points) Is  $P(c)$  a partition of  $V$ ? For each of the three conditions required to be a partition, explain why  $P(c)$  does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$$

always ☐sometimes ☐never ☐

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(7 points) Let  $f : \mathbb{Z}^+ \rightarrow \mathbb{P}(\mathbb{Z}^+)$  be defined by  $f(n) = \{p \in \mathbb{Z}^+ : n|p\}$ . For which natural numbers  $a$  and  $b$  is  $f(a)$  a subset of  $f(b)$ ? Briefly justify your answer.

(8 points) Check the (single) box that best characterizes each item.

If  $f : \mathbb{N} \rightarrow \mathbb{P}(\mathbb{Q})$   
then  $f(3)$  is

a rational ☐  
one or more rationals ☐

a set of rationals ☐  
a power set ☐

undefined ☐

$\{\{a, b\}, c\} = \{a, b, c\}$

true ☐

false ☐

Set  $B$  is a partition of a finite  
set  $A$ . Then

$|B| \leq 2^{|A|}$  ☐  
 $|B| = 2^{|A|}$  ☐

$|B| \leq |A|$  ☐  
 $|B| \leq |A + 1|$  ☐

$\binom{n}{1}$

-1 ☐

0 ☐

1 ☐

2 ☐

n ☐

undefined ☐

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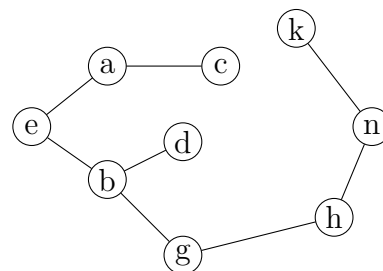
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Graph  $G$  is shown at right with set of nodes  $V$  and set of edges

$E$ . Let  $M : (V, \mathbb{N}) \rightarrow \mathbb{P}(V)$  be defined by

$M(x, n) = \{y \in V \mid \text{there is a path of length } n \text{ from } x \text{ to } y\}$ .

Let  $P(x) = \{M(x, n) \mid n \in \mathbb{N}\}$ .



(6 points) Give the value of  $M(g, n)$ , for all values of  $n$  from 0 to 3.

(7 points) Is  $P(g)$  a partition of  $V$ ? For each of the three conditions required to be a partition, explain why  $P(g)$  does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

 $\binom{n}{n}$ 

-1

☐

0

☐

1

☐

2

☐

n

☐

undefined

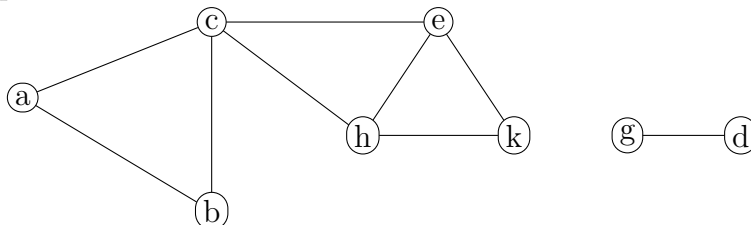
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Graph  $G$  is shown below with set of nodes  $V$  and set of edges  $E$ .

Let  $F : V \rightarrow \mathbb{P}(V)$  such that  $F(n) = \{v \in V \mid \text{there is a cycle containing } n \text{ and } v\}$ .  
 Let  $T = \{F(n) \mid n \in V\}$ .

(6 points) Fill in the following values:

$F(g) =$

$F(b) =$

$F(k) =$

(7 points) Is  $T$  a partition of  $V$ ? For each of the three conditions required to be a partition, explain why  $T$  does or doesn't satisfy that condition.

(2 points) Check the (single) box that best characterizes each item.

$\binom{n}{0}$

-1 ☐    0 ☐    1 ☐    2 ☐    n ☐    undefined ☐