Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

> John has a camera and there is a Meerkat m, such that m lives in New York and John has not photographed m

Solution: John does not have a camera or for every Meerkat m, m does not live in New York or John has photographed m

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

> For every violin v, if v is old or the maker of v is not known, then v is not valuable.

Solution: For every violin v, if v is valuable, then v is not old and the maker of v is known.

3. (5 points) Suppose that x is an integer and $x^2 + 3x - 18 < 0$. What are the possible values of x? Show your work.

Solution: $x^2 + 3x - 18 = (x + 6)(x - 3)$. So we have $(x + 6)(x - 3) < 0$. So one of $(x + 6)$ and $(x - 3)$ is negative and the other positive. Because $(x + 6)$ is larger, $(x + 6)$ must be the positive one.

So we have $x + 6 > 0$ and $x - 3 < 0$. So $x > -6$ and $x < 3$. Since x is an integer, it must be one of the following values:

$$-5, -4, -3, -2, -1, 0, 1, 2$$

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every Meerkat m, if m is in New York, then m is not in the wild or m is lost.

Solution: There is a Meerkat m, such that m is in New York, but m is in the wild and m is not lost.

2. (5 points) Solve $\frac{3}{x} + m = \frac{3}{p}$ for x, expressing your answer as a single fraction. Simplify your answer and show your work.

Solution: Multiplying by xp gives you $3p + mxp = 3x$.

So $3x - mxp = 3p$.

So $x(3 - mp) = 3p$.

So $x = \frac{3p}{3-mp}$.

3. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dinosaur d, if d is small and d is not a juvenile, then d is not a sauropod.

Solution: For every dinosaur d, if d is a sauropod, then d is not small or d is a juvenile.

Name:____

NetID:____________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Show that the following two expressions are not logically equivalent, by giving specific values of p, q for which they produce different values.

$p \rightarrow (q \rightarrow p)$

$(p \rightarrow q) \rightarrow p$

Solution: Set p and q to be false.

Then $p \rightarrow (q \rightarrow p)$ is true because its hypothesis is false.

$p \rightarrow q$ is also true. So $(p \rightarrow q) \rightarrow p$ is false because its hypothesis is true but its conclusion (p) is false.

A similar argument works if you set p to be false and q to be true.

2. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every cat c, if c is not fierce or c wears a collar, then c is a pet.

Solution: There exists a cat c that is either not fierce or wears a collar and is not a pet.

3. (5 points) Suppose that k is a positive integer, x is a positive real number, and $\frac{1}{k} = x + \frac{1}{6}$. What are the possible values for k? (Hint: k is an INTEGER.) Briefly explain or show work.

Solution: Observe that we can rearrange the equation as follows:

Since x is positive, $\frac{1}{k} = x + \frac{1}{6}$ implies that $\frac{1}{k} > \frac{1}{6}$. So k must be smaller than 6. But we were told that k was a positive integer. The only positive integers smaller than 6 are 1, 2, 3, 4, and 5.

Name:____

NetID:_________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

If it is raining, then there is a cyclist c such that c is getting wet.

Solution: It is raining and for every cyclist c, c is not getting wet.

2. (5 points) Describe all (real) solutions to the equation $2p^2 + p - 6 < 0$. Show your work.

Solution: $2p^2 + p - 6 = (2p - 3)(p + 2)$

So we have $(2p - 3)(p + 2) < 0$. Dividing by 2 gives us $(p - 1.5)(p + 2) < 0$.

$(p - 1.5)(p + 2)$ is negative when exactly one of the factors is positive. The positive factor must be $p + 2$ because it's larger. So we have $p + 2 > 0$, i.e. $p > -2$. And then also $p - 1.5 < 0$, i.e. $p < 1.5$.

So p is in the interval $(-2, 1.5)$.

3. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$(r \to q) \to r = r$

q	r	$r \to q$	$(r \to q) \to r$
T	T	T	T
T	F	T	F
F	T	F	T
F	F	T	F

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Give a truth table for the following expression and (using your truth table or other means) find a simpler expression equivalent to it.

Solution:

$(p \wedge q) \vee q = q$

p	q	$p \wedge q$	$(p \wedge q) \vee q$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	F

2. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every elephant e, if e likes to dance and e has good taste, then e likes Juluka.

Solution: For every elephant e, if e does not like Juluka, then e doesn't like to dance or e has bad taste.

3. (5 points) Solve $\frac{x}{2} - 1 < 3x + 9$ for x. (Assume x is real.) Show your work.

Solution: Multiplying both sides by 2 gives us $x - 2 < 6x + 18$. So $-20 < 5x$, and thus $x > -4$.

Name:____

NetID:________________________ **Lecture:** **A** **B**

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1. (5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every violin v, if v is old or the maker of v is not known, then v is not valuable.

Solution: There is a violin v, such that v is old or the maker of v is not known, but v is valuable.

2. (5 points) Suppose that f and g are functions whose inputs and outputs are real numbers, defined by $f(x) = x^2 - 1$ and $g(x) = \frac{x}{2}$. Compute the value of $g(f(y+1))$, showing your work.

Solution: $f(y+1) = (y+1)^2 - 1 = y^2 + 2y$

So $g(f(y+1)) = \frac{y^2+2y}{2}$

3. (5 points) State the contrapositive of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every garbage can c, if c was supplied by the city, then c is small or c has wheels.

Solution: For every garbage can c, if c is large and c does not have wheels, then c was not supplied by the city.