Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any real numbers x and y, if x or y is irrational, then xy is irrational.

   Solution: This is not true. Consider $x = y = \sqrt{2}$. Then x or y is irrational (because they are both irrational). But $xy = 2$ which is rational.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1012, 299)$. Show your work.

   Solution: $1012 - 3 \times 299 = 1012 - 897 = 115$

   $299 - 2 \times 115 = 299 - 230 = 69$

   $115 - 69 = 46$

   $69 - 46 = 23$

   $46 - 2 \times 23 = 0$

   So $\gcd(1012, 299) = 23$

3. (4 points) Check the (single) box that best characterizes each item.

$7 \mid -7$	true	☑	false	☐		
$k \equiv -k \pmod{k}$	always	☑	sometimes	☐	never	☐

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Claim: For all positive integers a, b, and c, if $\gcd(a, b) = 1$ and $\gcd(b, c) = 1$, then $\gcd(a, c) = 1$.

   Solution: This is false. Consider $a = c = 3$ and $b = 2$. Then a and b have no common factors, i.e. $\gcd(a, b) = 1$. Also b and c have no common factors, i.e. $\gcd(b, c) = 1$. But $\gcd(a, c) = 3$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(3927, 637)$. Show your work.

   Solution:

   $3927 - 6 \times 637 = 3927 - 3822 = 105$

   $637 - 6 \times 105 = 7$

   $105 - 15 \times 7 = 0$

   So the GCD is 7.

3. (4 points) Check the (single) box that best characterizes each item.

$\gcd(k, 0)$ for k non-zero	0 ☐	k ☑ √	undefined ☐
$7 \equiv 5 \pmod 1$	true ☑ √	false ☐	

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1. (5 points) Let a and b be integers, $b > 0$. We used two formulas to define the quotient q and the remainder r of a divided by b. One of these is $a = bq + r$. What is the other?

   Solution: $0 \leq r < b$

2. (6 points) Use the Euclidean algorithm to compute $\gcd(4263, 667)$. Show your work.

   Solution:

   $4263 - 6 \times 667 = 261$

   $667 - 2 \times 261 = 145$

   $261 - 145 = 116$

   $145 - 116 = 29$

   $116 - 4 \times 29 = 0$

   So the GCD is 29.

3. (4 points) Check the (single) box that best characterizes each item.

For all prime numbers p, there are exactly two natural numbers q such that $q \mid p$.	true ☑ $\surd$	false ☐
Zero is a factor of 7.	true ☐	false ☑ $\surd$

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integers a, b, q and r, if $a = bq + r$, then $\gcd(a, b) = \gcd(a, r)$.

   Solution: This is false. Consider $a = 18$, $b = 5$, $q = 3$, and $r = 3$. Then we have $18 = 5 \cdot 3 + 3$. $\gcd(a, b) = 1$, but $\gcd(a, r) = 3$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1568, 546)$. Show your work.

   Solution:

   $1568 - 546 \times 2 = 1568 - 1092 = 476$

   $546 - 476 = 70$

   $476 - 70 \times 6 = 476 - 420 = 56$

   $70 - 56 = 14$

   $56 - 14 \times 3 = 0$

   So the GCD is 14.

3. (4 points) Check the (single) box that best characterizes each item.

   If a and b are positive and $r = \text{remainder}(a, b)$, then $\gcd(b, r) = \gcd(b, a)$ — true [√] false [ ]

   $-7 \equiv 13 \pmod 6$ — true [ ] false [√]

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1. (5 points) Is the following claim true? Informally explain why it is false, or give a concrete example showing that it is not.

   There is an integer n such that $n \equiv 5 \pmod{6}$ and $n \equiv 2 \pmod{10}$?

   Solution: There is no such n. If $n \equiv 5 \pmod{6}$ and $n \equiv 2 \pmod{10}$, then $n = 5 + 6k$ and $n = 2 + 10j$, where k and j are integers. So $5 + 6k = 2 + 10j$. This implies that $3 = 10j - 6k$ which is impossible because the right side is divisible by 2 and the left side isn't.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(7917, 357)$. Show your work.

   Solution:

   $7917 - 22 \times 357 = 63$

   $357 - 5 \times 63 = 42$

   $63 - 42 = 21$

   $42 - 2 \times 21 = 0$

   So the GCD is 21.

3. (4 points) Check the (single) box that best characterizes each item.

$\gcd(0,0)$	0 ☐	k ☐	undefined ☑
$29 \equiv 2 \pmod{9}$	true ☑	false ☐	

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1. (5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

   For any positive integer k, $(k-1)^2 \equiv 1 \pmod{k}$.

   Solution: This is true. Notice that $(k-1) - (-1) = k$. So $k - 1 \equiv (-1) \pmod{k}$. Therefore $(k-1)^2 \equiv (-1)^2 \equiv 1 \pmod{k}$.

2. (6 points) Use the Euclidean algorithm to compute $\gcd(1183, 351)$. Show your work.

   Solution:

   $1183 - 3 \times 351 = 1183 - 1053 = 130$

   $351 - 2 \times 130 = 351 - 260 = 91$

   $130 - 91 = 39$

   $91 - 3 \times 39 = 91 - 78 = 13$

   $39 - 3 \times 13 = 0$

   So the GCD is 13.

3. (4 points) Check the (single) box that best characterizes each item.

   $k \equiv -k \pmod{7}$ always ☐ sometimes ☑ never ☐

   $-2 \equiv 2 \pmod{4}$ true ☑ false ☐