Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

$A = \{(x, y) \in \mathbb{R}^2 \ : \ y = x^2 - 4\}$

$B = \{(p, q) \in \mathbb{Z}^2 \ : q < 0\}$

$C = \{(a, b) \in \mathbb{R}^2 \ : \ |a| \leq 1\}$

Prove that $A \cap B \subseteq C$.

Solution: Proof: Let $(x, y) \in A \cap B$. Then, $(x, y) \in A$ and $(x, y) \in B$. So, from the definition of A, we know that $y = x^2 - 4$. From the definition of B, we know that $y < 0$ and that x and y are both integers.

$y = x^2 - 4 = (x - 2)(x + 2)$. So since $y < 0$, $-2 < x < 2$. But x is an integer. So the only possible values in this range are -1, 0, and 1. Therefore $|x| \leq 1$. So $(x, y) \in C$, which is what we needed to prove.

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$A = \{(x, y) \in \mathbb{Z}^2 \mid 2xy + 6y - 5x - 15 \geq 0\}$

$B = \{(a, b) \in \mathbb{Z}^2 \mid a \geq 0\}$

$C = \{(p, q) \in \mathbb{Z}^2 \mid q \geq 0\}$

Prove that $(A \cap B) \subseteq C$.

Solution: Suppose that (x, y) is an element of $(A \cap B)$. This means that (x, y) is an element of A and (x, y) is an element of B. So $2xy + 6y - 5x - 15 \geq 0$ and $x \geq 0$, by the definitions of A and B.

Notice that $2xy + 6y - 5x - 15 = (x + 3)(2y - 5)$. So $(x + 3)(2y - 5) \geq 0$. We know that $x + 3$ is positive because $x \geq 0$. So we must have $(2y - 5) \geq 0$.

Now, if $(2y - 5) \geq 0$, then $2y \geq 5$. So $y \geq \frac{5}{2}$. So $y \geq 0$. This means that (x, y) is an element of C which is what we needed to show.

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$A = \{\alpha(2,-4) + (1-\alpha)(-2,5)) \mid \alpha \in \mathbb{R}\}$

$B = \{(a,b) \in \mathbb{R}^2 \mid b \leq -1\}$

$C = \{(p,q) \in \mathbb{R}^2 \mid p \geq 0\}$

Prove that $A \cap B \subseteq C$.

Solution: Let (x,y) be a 2D point and suppose that $(x,y) \in A \cap B$. Then $(x,y) \in A$ and $(x,y) \in B$.

Since $(x,y) \in A$, $(x,y) = \alpha(2,-4) + (1-\alpha)(-2,5))$ where α is a real number. So $x = 2\alpha - 2(1-\alpha) = 4\alpha - 2$ And $y = -4\alpha + 5(1-\alpha) = 5 - 9\alpha$

Since $(x,y) \in B$, $y \leq -1$. So we have $y = 5 - 9\alpha \leq -1$. So $6 \leq 9\alpha$. So $\alpha \geq \frac{2}{3}$.

So then $x = 4\alpha - 2 \geq 4\frac{2}{3} - 2 = \frac{8}{3} - 2 = \frac{2}{3}$.

So $x \geq 0$ and therefore $(x,y) \in C$, which is what we needed to show.

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$A = \{a(1,0) + b(3,1) + c(2,4) \ : \ a, b, c \text{ are positive reals and } a + b + c = 1\}$

$B = \{(x, y) \in \mathbb{R}^2 \ : \ x \leq 3 \text{ and } y \geq 0\}$

Prove that $A \subseteq B$.

Solution: Let $(x, y) \in A$. By the definition of A, $(x, y) = a(1,0) + b(3,1) + c(2,4)$, where a, b, and c are positive reals and $a + b + c = 1$.

Then $(x, y) = (a + 3b + 2c, b + 4c)$. So $x = a + 3b + 2c$ and $y = b + 4c$.

We know that a, b, and c are positive, so $b + 4c$ must be positive. So $y \geq 0$.

Since a and c are positive and $a + b + c = 1$, we have

$$x = a + 3b + 2c \leq 3a + 3b + 3c = 3(a + b + c) = 3$$

So $y \geq 0$ and $x \leq 3$. Therefore (x, y) is in B, by the definition of the set B.

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$A = \{(x, y) \in \mathbb{R}^2 \ : \ xy \leq -7\}$

$B = \{(p^3, p^2) : p \in \mathbb{R}\}$

$C = \{(a, b) \in \mathbb{R}^2 \ : \ a < 0\}$

Prove that $A \cap B \subseteq C$.

Solution: Proof: Let $(x, y) \in A \cap B$. Then, $(x, y) \in A$ and $(x, y) \in B$. So, from the definition of A, we know that $xy \leq -7$. From the definition of B, we know that $x = p^3$ and $y = p^2$, for some real number p.

Since $xy \leq 7 < 0$, we know x and y have opposite signs and neither is zero. Since $y = p^2$, we know that y is positive. So x must be negative.

Since x is negative, $(x, y) \in C$, which is what we needed to prove.

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Prove that $A \cap B \subseteq C$.

Solution: Proof: Let $(x, y) \in A \cap B$. Then, $(x, y) \in A$ and $(x, y) \in B$. So, from the definition of A, we know that $y = x^2 - 4$. From the definition of B, we know that $y < 0$ and that x and y are both integers.

$y = x^2 - 4 = (x - 2)(x + 2)$. So since $y < 0$, $-2 < x < 2$. But x is an integer. So the only possible values in this range are -1, 0, and 1. Therefore $|x| \leq 1$. So $(x, y) \in C$, which is what we needed to prove.