Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{water}, \text{beer}, \text{wine}\}$ $B = \{\ \text{cup}, \text{mug}\ \}$ $C = \{\text{wine},\ (\text{water}, \text{beer})\ \}$

   $(A - C) \times B =$

   Solution: $\{\ (\text{water}, \text{cup}), (\text{beer}, \text{cup}), (\text{water}, \text{mug}), (\text{beer}, \text{mug}),\ \}$

   $A \cap B =$

   Solution: $\emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $\forall x \in \mathbb{N}$, if $x < -10$, then $x = \pi$.
   (π is the familiar constant.) true [√] false [ ] undefined [ ]

   $|A \times B| = |A| \times |B|$ true for all sets A [√] true for some sets A [ ] false for all sets A [ ]

3. (7 points) In $\mathbb{Z}_7$, find the value of $[3]^{37}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 6$.

   Solution: $[3]^2 = [9] = [2]$

   $[3]^4 = [2]^2 = [4]$

   $[3]^8 = [4]^2 = [16] = [2]$

   $[3]^{16} = [2]^2 = [4]$

   $[3]^{32} = [4]^2 = [2]$

   $[3]^{37} = [3]^{32} \cdot [3]^4 \cdot [3] = [2][4][3] = [24] = [3]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $(A - B) \cup (B - C) = (A \cup B) - (A \cap B \cap C)$

   Solution: This is not true. Suppose that A is the empty set, and $B = C = \{1\}$. Then 1 is in $(A \cup B)$ but not in $(A \cap B \cap C)$, So 1 is in $(A \cup B) - (A \cap B \cap C)$. However, 1 is neither in $A - B$ nor in $B - C$. So it's not in $(A - B) \cup (B - C)$

2. (4 points) Check the (single) box that best characterizes each item.

   $\emptyset \subseteq A$ true for all sets A [√] true for some sets A [ ] false for all sets A [ ]

   For any sets A and B, if $x \in A - B$, then $x \in A$. true [√] false [ ]

3. (7 points) In $\mathbb{Z}_9$, find the value of $[5]^{21}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 8$.

   Solution: $[5]^2 = [25] = [7]$

   $[5]^4 = [7]^2 = [49] = [4]$

   $[5]^8 = [4]^2 = [16] = [7]$

   $[5]^{16} = [7]^2 = [49] = [4]$

   $[5]^{21} = [5]^{16} \cdot [5]^4 \cdot [5] = [4][4][5] = [80] = [8]$

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $(A - B) - C = A - C$

   Solution: This is not true. Suppose that C is the empty set, and $A = B = \{1\}$. Then 1 is not $(A - B)$, therefore not in $(A - B) - C$. However, 1 is in $A - C$, because it's in A but not in C. So the two sets aren't equal.

2. (4 points) Check the (single) box that best characterizes each item.

   $A \times B = B \times A$ — true for all sets A and B ☐ — false for all sets A and B ☐ — true for some sets A and B ☑

   $\{\emptyset\} \times \{\emptyset\} =$ — $\emptyset$ ☐ — $\{\emptyset\}$ ☐ — $\{\emptyset, \emptyset\}$ ☐ — $\{(\emptyset, \emptyset)\}$ ☑

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[6]^{42}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[6]^2 = [36] = [3]$

   $[6]^4 = [3]^2 = [9]$

   $[6]^8 = [9]^2 = [81] = [4]$

   $[6]^{16} = [4]^2 = [16] = [5]$

   $[6]^{32} = [5]^2 = [25] = [3]$

   $[6]^{42} = [6]^{32} \cdot [6]^8 \cdot [6]^2 = [3][4][3] = [36] = [3]$

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{water}, \text{beer}, \text{wine}\}$ $B = \{\text{ cup}, \text{mug }\}$ $C = \{\text{wine}, \ (\text{water}, \text{beer})\ \}$

   $A \times (B \cap C) =$

   Solution: $A \times \emptyset = \emptyset$

   $|A \times B \times C| =$

   Solution: $3 \times 2 \times 2 = 12$

2. (4 points) Check the (single) box that best characterizes each item.

   If $x \in A \cap B$, then $x \in A$.

   true for all sets A and B [√] false for all sets A and B [ ] true for some sets A and B [ ]

   $\{13, 14, 15\} \times \emptyset =$

   $\emptyset$ [√] $\{\emptyset\}$ [ ] $\{13, 14, 15\}$ [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{12} + [9]^5$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = [5^2] = [25] = [3]$

   $[7]^8 = [3^2] = [9]$

   So $[7]^{12} = [7]^8 \cdot [7]^4 = [3] \cdot [9] = [27] = [5]$

   $[9]^2 = [81] = [4]$

   $[9]^4 = [4]^2 = [16] = 5$

   So $[9]^5 = [9] \cdot [5] = [45] = [1]$

   So $[7]^{12} + [9]^5 = [5] + [1] = [6]$.

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1. (4 points) Is this claim true? Give a concrete counter-example or briefly explain why it's true.

   For any sets A, B, and C, $A \times (B - C) = (A \times B) - (A \times C)$

   Solution: This is true. Elements in $A \times (B - C)$ need to have a first component from A and a second component that's in B but not in C. But these are the same conditions required for a element to be in $A \times B$ but not in $A \times C$.

2. (4 points) Check the (single) box that best characterizes each item.

   $\overline{A \cup B} = \overline{A} \cap \overline{B}$ true for all sets A [√] true for some sets A [ ] false for all sets A [ ]

   $\forall x \in \mathbb{R}$, if $\pi = 3$, then $x < 20$.
   (π is the familiar constant.) true [√] false [ ] undefined [ ]

3. (7 points) In $\mathbb{Z}_{13}$, find the value of $[7]^{18} + [7]^4$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 12$.

   Solution:

   $[7]^2 = [49] = [10] = [-3]$

   $[7]^4 = [-3]^2 = [9]$

   $[7]^6 = ([7]^2)^3 = [-3]^3 = [-27] = [-1]$

   $[7]^{18} = ([7]^6)^3 = [-1]^3 = [-1] = [12]$

   So $[7]^{18} + [7]^4 = [12] + [9] = [21] = [8]$

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (4 points) $A = \{\text{ginger}, \text{clove}, \text{nutmeg}\}$ $B = \{\text{ginger}, \text{vanilla}, \text{pepper}\}$ $C = \{\ (\text{clove}, \text{nutmeg})\ \}$

   $A \cap B =$

   Solution: $\{\text{ginger}\}$

   $A \cap C =$

   Solution: $\emptyset$

2. (4 points) Check the (single) box that best characterizes each item.

   $|A \cup B| \leq |A| + |B|$ true for all sets A [√] true for some sets A [ ] false for all sets A [ ]

   $\emptyset \times \emptyset =$ $\{\emptyset, \emptyset\}$ [ ] $\{\emptyset\}$ [ ] $\emptyset$ [√] undefined [ ]

3. (7 points) In $\mathbb{Z}_{11}$, find the value of $[7]^{38}$. You must show your work, keeping all numbers in your calculations small. **You may not use a calculator.** You must express your final answer as $[n]$, where $0 \leq n \leq 10$.

   Solution:

   $[7]^2 = [49] = [5]$

   $[7]^4 = ([7]^2)^2 = [5]^2 = [25] = [3]$

   $[7]^8 = ([7]^4)^2 = [3]^2 = [9] = [-2]$

   $[7]^16 = ([7]^8)^2 = [-2]^2 = [4]$

   $[7]^32 = ([7]^{16})^2 = [4]^2 = [16] = [5]$

   $[7]^{38} = [7]^{32} \cdot [7]^4 \cdot [7]^2 = [5] \cdot [3] \cdot [5] = [15] \cdot [5] = [4] \cdot [5] = [20] = [9]$