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Suppose that T is a relation on the integers which is transitive. Let's define a relation R on pairs of integers such that $(p,q)R(a,b)$ if and only if $(a+b)T(p+q)$. Prove that R is transitive.

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Let T be the relation defined on $\mathbb{Z}^2$ by

$(x, y)T(p, q)$ if and only if $x < p$ or $(x = p$ and $y \leq q)$

Prove that T is antisymmetric.

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Let $A = \{(a, b) \in \mathbb{R}^2 \ : \ a > 0, b > 0, \text{ and } a + b = 90\}$. That is, an element of A is a pair of positive reals that sum to 90 (e.g. the non-right angles in a right triangle). Now, let's define a relation $\sim$ on A as follows:

$(a, b) \sim (p, q)$ if and only if $a = p$ or $a = q$.

Prove that R is transitive.

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For any two real numbers with $a \leq b$, the closed interval $[a, b]$ is defined by $[a, b] = \{x \in \mathbb{R} \ : \ a \leq x \leq b\}$. Let J be the set containing all closed intervals $[a, b]$. Let's define the relation F on J as follows:

$[s, t]F[p, q]$ if and only if $q \leq s$

Prove that F is antisymmetric.

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Let $A = \mathbb{Z}^+ \times \mathbb{Z}^+$, i.e. pairs of positive integers. Consider the relation T on A defined by

$(x, y)T(p, q)$ if and only if $x \leq p$ and $xy \leq pq$

Prove that T is antisymmetric.

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A closed interval of the real line can be represented as a pair (c, r), where c is the center of the interval and r is its radius. Let $X = \{(c, r) \mid c, r \in \mathbb{R}, r \geq 0\}$ be the set of closed intervals represented this way.

Now, let's define the interval "starts" relation $\preceq$ on X as follows

$(c, r) \preceq (d, q)$ if and only if $c + q = d + r$ and $c + r \leq d + q$

Prove that $\preceq$ is transitive.