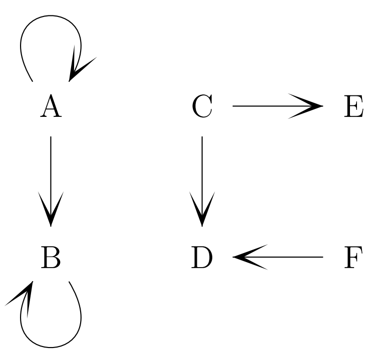
Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☐
Symmetric:	☐	Antisymmetric:	√
Transitive:	√		

2. (5 points) Recall that $\mathbb{Z}^2$ is the set of all pairs of integers. Let's define the equivalence relation $\sim$ on $\mathbb{Z}^2$ as follows: $(a, b) \sim (p, q)$ if and only $ab = pq$. List three members of $[(5, 6)]$.

   Solution: (5,6), (1,30), (-15,-2)

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x \leq p$ and $y \leq q$. Is T antisymmetric? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is antisymmetric. Suppose that $(x, y)T(p, q)$ and $(p, q)T(x, y)$. Then $x \leq p$ and $y \leq q$, and also $p \leq x$ and $q \leq y$. So $x = y$ and $y = q$. So $(x, y) = (p, q)$.

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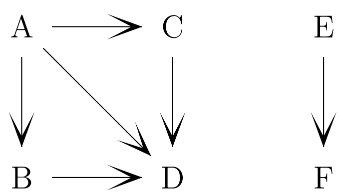

Reflexive:	☐	Irreflexive:	$\surd$
Symmetric:	☐	Antisymmetric:	$\surd$
Transitive:	$\surd$		

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be antisymmetric.

   Solution: For any $x, y \in A$, if xRy and yRx, then $x = y$. Or for any $x, y \in A$, if xRy and $x \neq y$, then $y \not R x$.

3. (5 points) Suppose that R is an equivalence relation on the integers. Is it true that $y \in [x]_R$ if and only if $x \in [y]_R$, for any integers x and y? Informally explain why it's true or give a concrete counter-example.

   Solution: This is true. If $y \in [x]_R$, then yRx. But equivalence relations are symmetric, so this implies that xRy, which means that $x \in [y]_R$. The same argument also works in the opposite direction.

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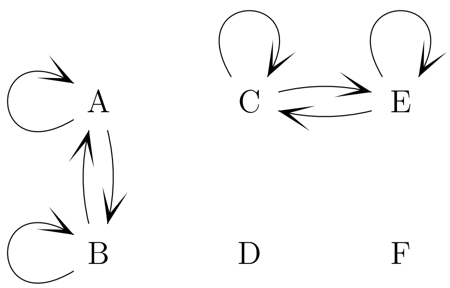
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☐
Symmetric:	☑ √	Antisymmetric:	☐
Transitive:	☑ √		

2. (5 points) Suppose that R is a relation on a set A. Using precise mathematical words and notation, define what it means for R to be symmetric.

   Solution: For any $x, y \in A$, if xRy, then yRx.

3. (5 points) Suppose that R is the relation on the set of integers such that aRb if and only if $|a - b| \leq 13$ Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: No, it is not transitive. Consider $a = 0$, $b = 13$, $c = 26$. Then aRb and bRc. However, $|a - c| = 26$, so $a \not R c$.

Name:____

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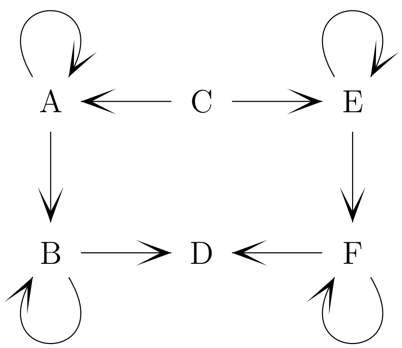
Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive:	☐	Irreflexive:	☐
Symmetric:	☐	Antisymmetric:	☑ √
Transitive:	☐		

2. (5 points) Suppose that S is the set of all binary strings (i.e. finite sequences of 1's and 0's). Suppose that $\sim$ is the relation on S where $a \sim b$ if and only if a and b contain the same number of 1's. For example, `0101` $\sim$ `1000001`. List three members of [`111`].

Solution: For example, `111`, `1101`, and `01110`.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $x - p \leq y - q$. Is T transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

Solution: This relation is transitive. Suppose that $(x, y)T(p, q)$ and $(p, q)T(m, n)$. Then $x - p \leq y - q$ and also $p - m \leq q - n$. Adding these two equations together, we get $(x - p) + (p - m) \leq (y - q) + (q - n)$. This simplifies to $x - m \leq y - n$. So $(x, y)T(m, n)$.

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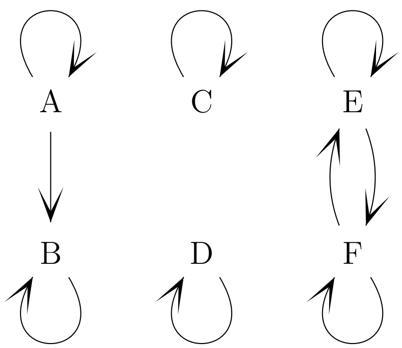
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: [√] Irreflexive: [ ]

Symmetric: [ ] Antisymmetric: [ ]

Transitive: [√]

2. (5 points) Let's define the equivalence relation $\sim$ on $\mathbb{R}$ sucn that $x \sim y$ if and only $|x - y| \in \mathbb{Z}$. List three members of $[1.7]$.

   Solution: For example, 1.7, 2.7, and 1009.7.

3. (5 points) Let T be the relation defined on set of pairs $(x, y) \in \mathbb{R}^2$ such that $(x, y)T(p, q)$ if and only if $xp + yq = 0$. Is T irreflexive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: This relation is not irreflexive, because $(0, 0)$ is related to itself.

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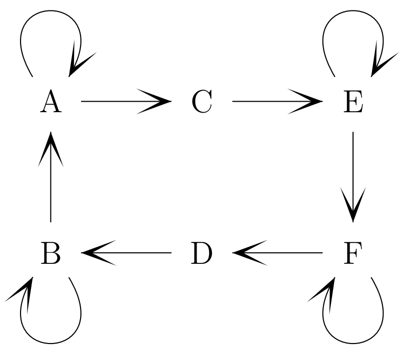
1. (5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☑ √

Transitive: ☐

2. (5 points) Can a relation be reflexive, symmetric, and also antisymmetric? Either give such a relation or briefly explain why it's not possible to construct one.

   Solution: Yes, this is possible. Suppose that our set contains three elements a, b, and c. And suppose that we have aRa, bRb, cRc and no other relationships. Then R is reflexive, symmetric and also antisymmetric.

3. (5 points) Let R be the relation on $\mathbb{Z}$ such that xRy if and only if $|x| + |y| = 2$

   Is R transitive? Informally explain why it is, or give a concrete counter-example showing that it is not.

   Solution: No, R is not transitive. We have $0R2$ and $2R0$, but not $0R0$.