Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) Suppose that $f : \mathbb{Z} \to \mathbb{Z}$ is onto. Let's define $g : \mathbb{Z}^2 \to \mathbb{Z}$ by $g(x, y) = (1 - f(x))f(y)$. Prove that g is onto.

2. (5 points) Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ which is one-to-one but not onto. Be specific.

Name:____

NetID:______________________ **Lecture:** **A** **B**

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1. (10 points) Let's define the function $f : \mathbb{R}^+ \to \mathbb{R}$ by $f(x) = \frac{x^2+2}{3x^2}$. ($\mathbb{R}^+$ is the positive reals.) Prove that f is one to one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "onto." You must use explicit quantifiers. Do not assume the reader knows what the image of the function is.

Name:____

NetID:______________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) Let P be the set of pairs of positive integers. Suppose that $f : P \to \mathbb{R}^2$ is defined by $f(x, y) = (\frac{x}{y}, x + y)$. Prove that f is one-to-one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) the behavior of increasing functions.

2. (5 points) $A = \{2, 3, 4, 5, 6, 7, 8 \ldots\}$, i.e. the integers ≥ 2

   $B = \{1, 2, 4, 8, 16, 32, 64, \ldots\}$, i.e. powers of 2 starting with 1

   Give a specific formula for a bijection $f : A \to B$. (You do not need to prove that it is a bijection.)

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is onto. Let's define $g : \mathbb{N}^2 \to \mathbb{Z}$ by $g(m, n) = (2 - n)f(m)$. Prove that g is onto.

2. (5 points) Suppose that $g : A \to B$ and $f : B \to C$. Prof. Snape claims that if g is onto, then $f \circ g$ is onto. Disprove this claim using a concrete counter-example in which A, B, and C are all small finite sets.

Name:____

NetID:________________________ **Lecture:** **A** **B**

Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

1. (10 points) Let $g : \mathbb{N} \to \mathbb{N}$ be onto, and let $f : \mathbb{N}^2 \to \mathbb{Z}$ be defined by $f(n, m) = (m-1)g(n)$. Prove that f is onto.

2. (5 points) Using precise mathematical words and notation, define what it means for a function $g : M \to C$ to be "one-to-one." You must use explicit quantifiers; do not use words like "unique".

Name:____

NetID:________________________ **Lecture:** **A** **B**

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1. (10 points) Let's define the function $f : \mathbb{R}^+ \to \mathbb{R}$ by $f(x) = \frac{4x-1}{2x+5}$. ($\mathbb{R}^+$ is the positive reals.) Prove that f is one to one. You must work directly from the definition of one-to-one. Do not use any facts about (for example) derivatives or the behavior of increasing functions.

2. (5 points) Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ which is onto but not one-to-one. Be specific.