Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = p$ and $|B| = q$, $p \leq q$. How many different one-to-one functions are there from A to B?

   Solution: $\frac{q!}{(q-p)!}$

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = |x|$ then $\mathbb{N}$ is the ____ of f. domain ☐ co-domain ☐ image ☑

   $g : \mathbb{R}^2 \to \mathbb{R}$, $g(x, y) = \lfloor x \rfloor + y$ onto ☑ not onto ☐ not a function ☐

   $g : \mathbb{R}^2 \to \mathbb{R}^2$, $g(x, y) = (y, 3x)$ one-to-one ☑ not one-to-one ☐ not a function ☐

   Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, every color appears on at least two vertices. true ☐ false ☑

   $\forall x \in \mathbb{Q},\ \exists m, n \in \mathbb{Z},\ x = \frac{m}{n}$ true ☑ false ☐

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Xin plans to randomly draw a hand of cards from a standard deck of 52 cards (evenly divided among 4 suits). He'd like to be sure the hand includes 3 cards with the same suit. How large must the hand be? Briefly justify your answer.

   Solution: He needs to draw 9 cards.

   Suppose the hand contained ≤ 2 cards from each suit. Since there are four suits, that means the total number of cards must be ≤ 8. Therefore, 9 cards guarantees 3 with the same suit.

2. (10 points) Check the (single) box that best characterizes each item.

   A function is one-to-one if and only if each value in the domain has exactly one image. true ☐ false ☑

   $g : \mathbb{R}^2 \to \mathbb{R}$, $g(x, y) = \lfloor x \rfloor + y$ one-to-one ☐ not one-to-one ☑ not a function ☐

   $g : \mathbb{R} \to [-1, 1]$, $g(x) = \sin(x)$ onto ☑ not onto ☐ not a function ☐

   $f : \mathbb{N}^2 \to \mathbb{Z}$, $f(p, q) = 2^p 3^q$ one-to-one ☑ not one-to-one ☐ not a function ☐

   $\forall x \in \mathbb{Z}^+,\ \exists y \in \mathbb{Z}^+,\ xy = 1$ true ☐ false ☑

Name:_____

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = 50$ and $B = \{5, 6\}$. How many onto functions are there from A to B? Briefly justify or show work. (Hint: how many non-onto functions are there?)

   Solution: There's only two ways to create a function from A to B that is not onto: all input values map to 5 or all input values map to 6. The total number of functions from A to B is 2^{50}. So the total number of onto functions is $2^{50} - 2$.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{N} \to \mathbb{Z}$ is a function such that $f(x) = -|x|$ then $\mathbb{N}$ is the ____ of f. — domain [√] co-domain [ ] image [ ]

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x|x|$ — onto [ ] not onto [√] not a function [ ]

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \lfloor \frac{x}{3} \rfloor$ — one-to-one [ ] not one-to-one [√] not a function [ ]

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x|x|$ — one-to-one [√] not one-to-one [ ] not a function [ ]

   $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x - y < 100$ — true [√] false [ ]

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ``minimalist''? Show your work.

   Solution: There are 10 characters total, with two copies of m and three copies of i. So the total number of permutations is

   $$\frac{10!}{3!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

   If a function is onto, then each value in the co-domain has at least one pre-image. true [√] false [ ]

   $g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$, $g(x, y) = \gcd(x, y)$ one-to-one [ ] not one-to-one [√] not a function [ ]

   $g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$, $g(x, y) = \gcd(x, y)$ onto [√] not onto [ ] not a function [ ]

   $f : \mathbb{R} \to \mathbb{Z}$, $f(x) = x$ one-to-one [ ] not one-to-one [ ] not a function [√]

   $\exists m, n \in \mathbb{Z},\ \forall x \in \mathbb{Q},\ x = \frac{m}{n}$ true [ ] false [√]

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that A is a set containing $k+1$ (distinct) integers. Use the Pigeonhole Principle to show that there are x and y in A ($x \neq y$) such that $x - y$ is a multiple of k.

   Solution: (You don't need to be this formal for full credit.) Let $A = \{x_1, x_2, \ldots, x_{k+1}\}$. We can represent each integer x_i in terms of its quotient and remainder mod k, i.e. $x_i = kq_i + r_i$, where $0 \leq r_i < k$. There are k possible remainders, but $k+1$ numbers in A. So two numbers must share the same remainder, which implies that they differ by a multiple of k.

2. (10 points) Check the (single) box that best characterizes each item.

   A function is one-to-one if and only if each value in the co-domain has at most one pre-image. true [√] false [ ]

   $g : \mathbb{Z}^2 \to \mathbb{Z}^2$, $g(x, y) = (y, 3x)$ one-to-one [√] not one-to-one [ ] not a function [ ]

   $g : \mathbb{Z} \to \mathbb{N}$, $g(x) = x$ one-to-one [ ] not one-to-one [ ] not a function [√]

   $g : \mathbb{N}^2 \to \mathbb{N}$, $g(x, y) = \gcd(x, y)$ onto [ ] not onto [ ] not a function [√]

   $\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ x - y < 100$ true [ ] false [√]

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ``tattletale''? Show your work.

   Solution: There are 10 letters total, with 4 copies of t, two a's, two e's, and two l's. So the total number of possibilities is

   $$\frac{10!}{4!2!2!2!}$$

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the real numbers is the _____ of f.
   domain ☐ co-domain ☑ image ☐

   $g : \mathbb{N}^2 \to \mathbb{N}$, $g(x, y) = \gcd(x, y)$
   one-to-one ☐ not one-to-one ☐ not a function ☑

   $f : \mathbb{N}^2 \to \mathbb{N}$, $f(p, q) = pq$
   onto ☑ not onto ☐ not a function ☐

   Each dorm room is given an access code between 1 and 10 (inclusive). According to the pigeonhole principle, if there are 21 dorm rooms, then every access code must be shared by at least two rooms.
   true ☐ false ☑

   $\forall m, n \in \mathbb{Z},\ \exists x \in \mathbb{Q},\ x = \frac{m}{n}$
   true ☐ false ☑