Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = p$ and $|B| = q$, $p \leq q$. How many different one-to-one functions are there from A to B?

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = |x|$ then $\mathbb{N}$ is the ____ of f. domain ☐ co-domain ☐ image ☐

   $g : \mathbb{R}^2 \to \mathbb{R}$
   $g(x, y) = \lfloor x \rfloor + y$ onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{R}^2 \to \mathbb{R}^2$
   $g(x, y) = (y, 3x)$ one-to-one ☐ not one-to-one ☐ not a function ☐

   Suppose a graph with 12 vertices is colored with exactly 5 colors. By the pigeonhole principle, every color appears on at least two vertices. true ☐ false ☐

   $\forall x \in \mathbb{Q},\ \exists m, n \in \mathbb{Z},\ x = \frac{m}{n}$ true ☐ false ☐

Name:_____

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Xin plans to randomly draw a hand of cards from a standard deck of 52 cards (evenly divided among 4 suits). He'd like to be sure the hand includes 3 cards with the same suit. How large must the hand be? Briefly justify your answer.

2. (10 points) Check the (single) box that best characterizes each item.

   A function is one-to-one if and only if each value in the domain has exactly one image. true ☐ false ☐

   $g : \mathbb{R}^2 \to \mathbb{R}$
   $g(x, y) = \lfloor x \rfloor + y$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $g : \mathbb{R} \to [-1, 1]$
   $g(x) = \sin(x)$ onto ☐ not onto ☐ not a function ☐

   $f : \mathbb{N}^2 \to \mathbb{Z}$
   $f(p, q) = 2^p 3^q$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $\forall x \in \mathbb{Z}^+,\ \exists y \in \mathbb{Z}^+,\ xy = 1$ true ☐ false ☐

Name:_____

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that $|A| = 50$ and $B = \{5, 6\}$. How many onto functions are there from A to B? Briefly justify or show work. (Hint: how many non-onto functions are there?)

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{N} \to \mathbb{Z}$ is a function such that $f(x) = -|x|$ then $\mathbb{N}$ is the ____ of f. domain ☐ co-domain ☐ image ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x|x|$ onto ☐ not onto ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = 7 - \lfloor \frac{x}{3} \rfloor$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $g : \mathbb{Z} \to \mathbb{Z}$, $g(x) = x|x|$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $\forall x \in \mathbb{Z},\ \exists y \in \mathbb{Z},\ x - y < 100$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ``minimalist''? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

If a function is onto, then each value in the co-domain has at least one pre-image. true ☐ false ☐

$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$
$g(x, y) = \gcd(x, y)$ one-to-one ☐ not one-to-one ☐ not a function ☐

$g : (\mathbb{Z}^+)^2 \to \mathbb{Z}^+$
$g(x, y) = \gcd(x, y)$ onto ☐ not onto ☐ not a function ☐

$f : \mathbb{R} \to \mathbb{Z}$
$f(x) = x$ one-to-one ☐ not one-to-one ☐ not a function ☐

$\exists m, n \in \mathbb{Z},\ \forall x \in \mathbb{Q},\ x = \frac{m}{n}$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) Suppose that A is a set containing $k+1$ (distinct) integers. Use the Pigeonhole Principle to show that there are x and y in A ($x \neq y$) such that $x - y$ is a multiple of k.

2. (10 points) Check the (single) box that best characterizes each item.

A function is one-to-one if and only if each value in the co-domain has at most one pre-image. true ☐ false ☐

$g : \mathbb{Z}^2 \to \mathbb{Z}^2$
$g(x, y) = (y, 3x)$ one-to-one ☐ not one-to-one ☐ not a function ☐

$g : \mathbb{Z} \to \mathbb{N}$
$g(x) = x$ one-to-one ☐ not one-to-one ☐ not a function ☐

$g : \mathbb{N}^2 \to \mathbb{N}$
$g(x, y) = \gcd(x, y)$ onto ☐ not onto ☐ not a function ☐

$\exists y \in \mathbb{Z},\ \forall x \in \mathbb{Z},\ x - y < 100$ true ☐ false ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (5 points) How many different 10-letter strings can be made be rearranging the characters in the word ``tattletale''? Show your work.

2. (10 points) Check the (single) box that best characterizes each item.

   If $f : \mathbb{Z} \to \mathbb{R}$ is a function such that $f(x) = 2x$ then the real numbers is the ____ of f. domain ☐ co-domain ☐ image ☐

   $g : \mathbb{N}^2 \to \mathbb{N}$
   $g(x, y) = \gcd(x, y)$ one-to-one ☐ not one-to-one ☐ not a function ☐

   $f : \mathbb{N}^2 \to \mathbb{N}$
   $f(p, q) = pq$ onto ☐ not onto ☐ not a function ☐

   Each dorm room is given an access code between 1 and 10 (inclusive). According to the pigeonhole principle, if there are 21 dorm rooms, then every access code must be shared by at least two rooms. true ☐ false ☐

   $\forall m, n \in \mathbb{Z},\ \exists x \in \mathbb{Q},\ x = \frac{m}{n}$ true ☐ false ☐