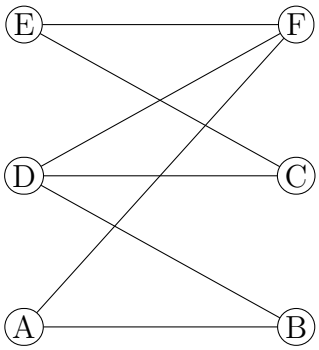
Name:_____

NetID:_____ **Lecture: A B**

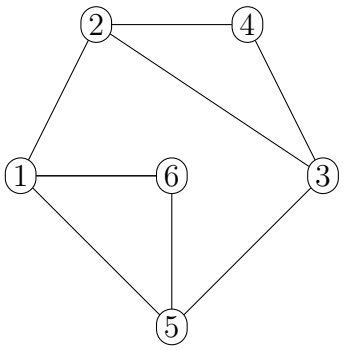
Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (10 points) Are graphs X and Y (below) isomorphic? Justify your answer.

Graph X



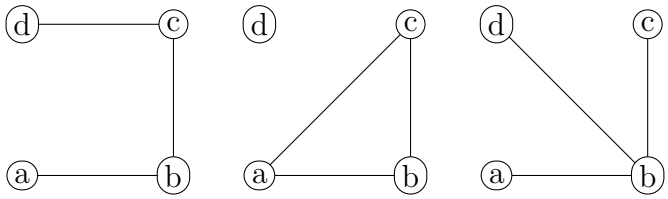
Graph Y



Solution: No, they are not isomorphic. Graph X has 7 edges but graph Y has 8 edges. [And various other features fail to match as well.]

2. (5 points) Show three graphs, each with exactly four nodes and three edges, none of which are isomorphic.

Solution:



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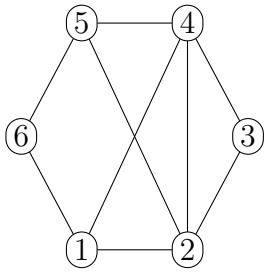
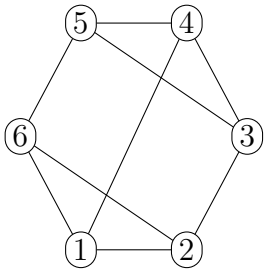


Graph Y



Solution: No, they are not isomorphic. Graph X has two nodes with degree 4. All the nodes in Graph Y have degree 3.

2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Is it possible to construct a graph with degree sequence: 5, 3, 2, 2, 2, 0? Show how or briefly explain why this isn't possible.

Solution: This isn't possible. Since one of the six nodes has degree 5, it's connected to all the other nodes. But then we can't have a node with degree 0.

Name:_____

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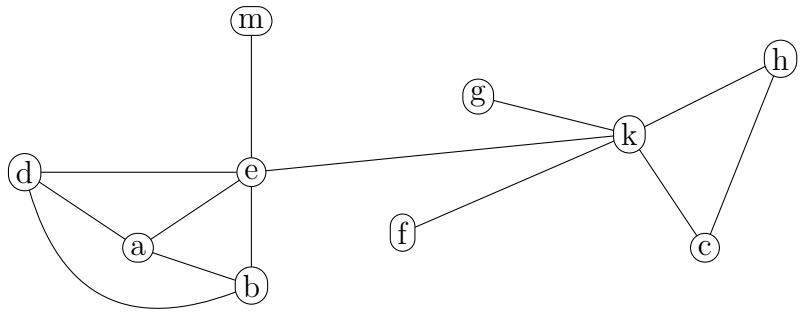
Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (10 points) How many isomorphisms are there from G (below) to itself? Justify your answer and/or show your work clearly .

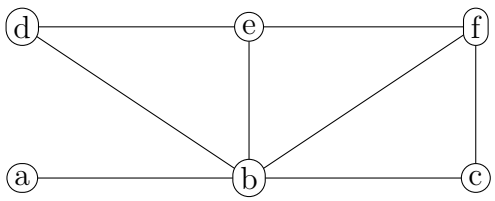


Solution: Nodes g and f can be swapped (2 choices). Nodes h and c can be swapped (2 choices). The nodes a, b, and d can be permuted (3! choices). So there are a total of $2 \cdot 2 \cdot 3! = 24$ isomorphisms.

2. (5 points) The degree sequence of a graph is the list of the degrees of all the nodes in the graph, arranged in numerical order, largest to smallest. Is it possible to construct a graph with degree sequence: 5, 3, 3, 2, 2, 1? Show how or briefly explain why this isn't possible.

Solution:

Yes. Here's a picture.



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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

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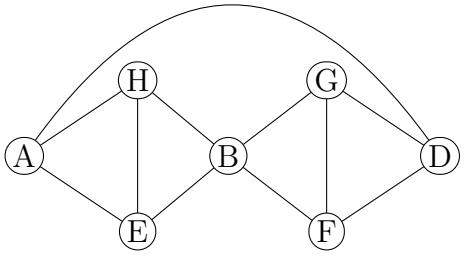
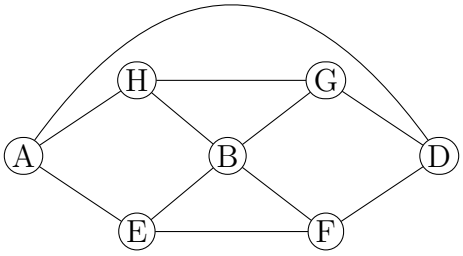
Graph X



Graph Y



Solution: No, they are not isomorphic. Graph X contains four 3-cycles but Graph X contains only two 3-cycles.

2. (5 points) What is the difference between a cycle and a closed walk?

Solution: A cycle uses each node only once, except that the first and last nodes are the same. Also, a cycle must contain at least three nodes.

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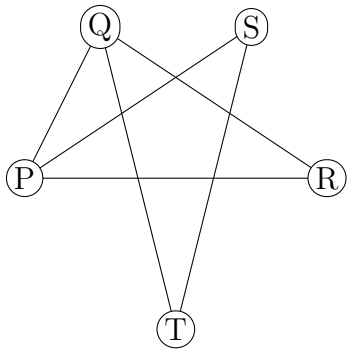
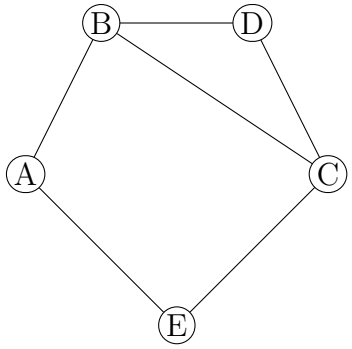


Graph Y



Solution: Yes, they are isomorphic. For example, use the map f, where f(R) = D, f(Q) = B, f(P) = C, f(S) = E, and f(T) = A.

2. (5 points) If G is a graph, its complement G' has the same nodes as G but G' has an edge between nodes x and y if and only G does not have an edge between x and y. Give a succinct high-level description of the complement of $K_{2,3}$. Briefly justify or show work.

Solution: Let's label the nodes a, b, c, d, and e, where a and b form one component of the bipartite graph and c, d, and e form the other. Then the complement contains all edges within a single component: ab, ad, de, and ec. So the complement consists of a 3-cycle, plus two nodes joined by an edge.

[A picture would also be a good way to justify your answer.]

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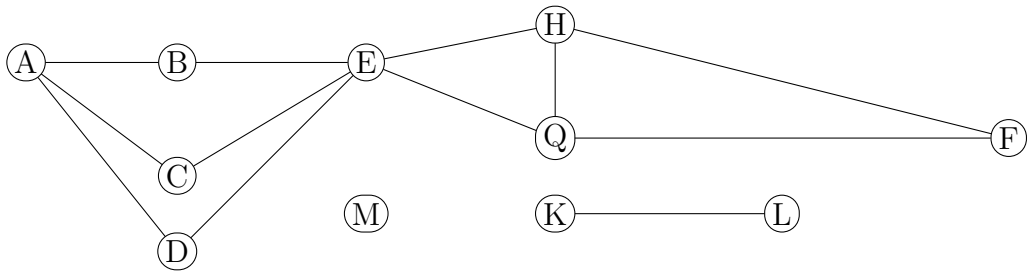

Solution: Node E must map to itself. Nodes B, C, and D can be permuted (3! choices). Nodes H and Q can be swapped (2 choices), as can nodes K and L (2 choices). So there are $2 \cdot 2 \cdot 3! = 24$ isomorphisms from the graph to itself.

2. (5 points) What is the diameter of C_n?

Solution: The diameter of C_n is $n/2$ if n is even, and $(n-1)/2$ if n is odd.