

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    10    11    12    1    2    3    4    5    6

Use (strong) induction to prove the following claim:

$$\text{Claim: } \sum_{p=1}^n \frac{2}{p^2+2p} = \frac{3}{2} - \frac{1}{(n+1)} - \frac{1}{(n+2)}$$

Proof by induction on  $n$ .

Base case(s):

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:\_\_\_\_\_

NetID:\_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    10    11    12    1    2    3    4    5    6

If  $f$  is a function, recall that  $f'$  is its derivative. Recall the product rule: if  $f(x) = g(x)h(x)$ , then  $f'(x) = g'(x)h(x) + g(x)h'(x)$ . Assume we know that the derivative of  $f(x) = x$  is  $f'(x) = 1$ .

Use (strong) induction to prove the following claim:

For any positive integer  $n$ , if  $f(x) = x^n$  then  $f'(x) = nx^{n-1}$ .

Proof by induction on  $n$ .

**Base case(s):**

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step:**

Name:\_\_\_\_\_

NetID:\_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    10    11    12    1    2    3    4    5    6

Let  $A$  be a constant integer. Use (strong) induction to prove the following claim. Remember that  $0! = 1$ .

Claim: For any integer  $n \geq A$ ,  $\sum_{p=A}^n \frac{p!}{A!(p-A)!} = \frac{(n+1)!}{(A+1)!(n-A)!}$

Proof by induction on  $n$ .

**Base case(s):**

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step:**

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    10    11    12    1    2    3    4    5    6

Use (strong) induction to prove the following claim:

Claim:  $\sum_{p=1}^n 2(-1)^p p^2 = (-1)^n n(n+1)$ , for all positive integers  $n$

Proof by induction on  $n$ .

**Base case(s):**

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step:**

Name:\_\_\_\_\_

NetID:\_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    10    11    12    1    2    3    4    5    6

Use (strong) induction and the fact that  $\sum_{i=0}^n i = \frac{n(n+1)}{2}$  to prove the following claim:

For all natural numbers  $n$ ,  $(\sum_{i=0}^n i)^2 = \sum_{i=0}^n i^3$

Proof by induction on  $n$ .

**Base case(s):**

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step:** (Start by removing the top term from the sum on the lefthand side.)

Name: \_\_\_\_\_

NetID: \_\_\_\_\_ Lecture:    A    B

Discussion:    Thursday    Friday    10    11    12    1    2    3    4    5    6

The operator  $\prod$  is like  $\sum$  except that it multiplies its terms rather than adding them. So e.g.  $\prod_{p=3}^5 (p+1) = 4 \cdot 5 \cdot 6$ . Use (strong) induction to prove the following claim:

$$\prod_{p=2}^n \left(1 - \frac{1}{p^2}\right) = \frac{n+1}{2n} \text{ for any integer } n \geq 2.$$

Proof by induction on  $n$ .

**Base case(s):**

**Inductive Hypothesis** [Be specific, don't just refer to "the claim"]:

**Rest of the inductive step:**