

Name: _____

NetID: _____ Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (11 points) Let's define two sets as follows:

$$A = \{(x, y) \in \mathbb{R}^2 : y = 3x + 7\}$$

$$B = \{\lambda(-2, 1) + (1 - \lambda)(1, 10) : \lambda \in \mathbb{R}\}$$

Prove that $A = B$ by proving two subset inclusions.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=0}^n \frac{1}{2^k} \quad 1 - \left(\frac{1}{2}\right)^{n-1} \quad \square \quad 2 - \left(\frac{1}{2}\right)^n \quad \square \quad 1 - \left(\frac{1}{2}\right)^n \quad \square \quad 2 - \left(\frac{1}{2}\right)^{n-1} \quad \square$$

$$\text{Chromatic number of } C_n. \quad 2 \quad \square \quad 3 \quad \square \quad \leq 3 \quad \square \quad \leq 4 \quad \square$$

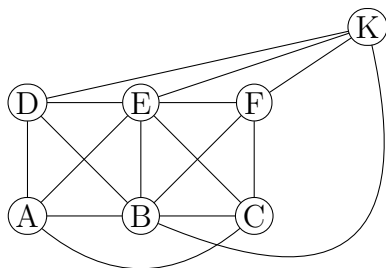
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1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{k=1}^n k! \quad \sum_{p=0}^{n+1} (p+1)! \quad \sum_{k=0}^{n+1} (k-1)! \quad \sum_{k=0}^{n-1} (k+1)! \quad \sum_{p=0}^{n+1} k!$$

All elements of M are also elements of X .

$$M = X \quad M \subseteq X \quad X \subseteq M$$

Chromatic number of G

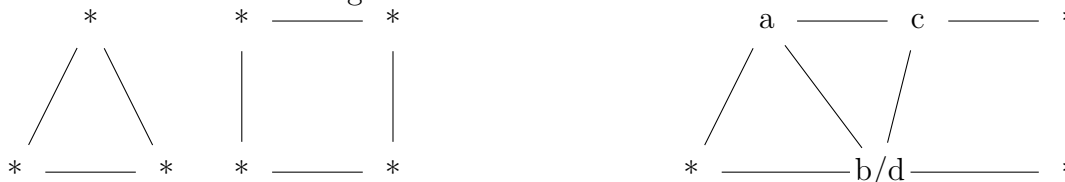
$$\mathcal{C}(G) \quad \phi(G) \quad \chi(G) \quad \|G\|$$

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1. (11 points) Recall that if G is a graph, then $\chi(G)$ is its chromatic number. Suppose that G is a graph and H is another graph not connected to G . Suppose G and H each have at least two nodes and at least one edge. Dr. Evil picks two adjacent nodes a and b from G , and also two adjacent nodes c and d from H . He merges G and H into a single graph T by merging b and d into a single node, and adding an edge connecting a and c . So, if G and H are as shown on the left, then T might look as shown on the right.



Describe how $\chi(T)$ is related to $\chi(G)$ and $\chi(H)$, justifying your answer.

2. (4 points) Check the (single) box that best characterizes each item.

$$\sum_{k=-2}^n k^2 \quad \sum_{p=0}^{n+2} (p+2)^2 \quad \sum_{p=0}^{n-2} (p-2)^2 \quad \sum_{p=0}^{n+2} (p-2)^2 \quad \sum_{p=0}^{n+2} p^2$$

W_7 is a subgraph of graph H . 4 is _____ the chromatic number of H .

an upper bound on _____
a lower bound on _____

exactly

not a bound on

☐
☐
☐
☐

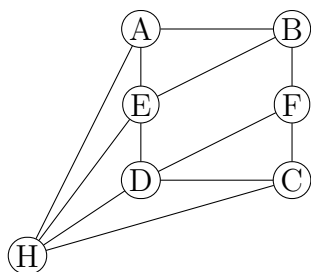
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1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{i=0}^{k-1} (k \cdot i + 2)$$

$$\frac{k^2(k+1)}{2} + 2k$$

☐

$$\frac{k(k+1)}{2} + 2(k-1)$$

☐

$$\frac{k^2(k-1)}{2} + 2k$$

☐

$$\frac{k(k-1)}{2} + 2(k-1)$$

☐

When I poured 5 gallons of water into the bucket, some spilled over the top. 5 gallons is _____ how much the bucket holds.

an upper bound on

☐

exactly

☐

a lower bound on

☐

not a bound on

☐

Chromatic number of a bipartite graph with at least two vertices.

1

☐

2

☐

3

☐

can't tell

☐

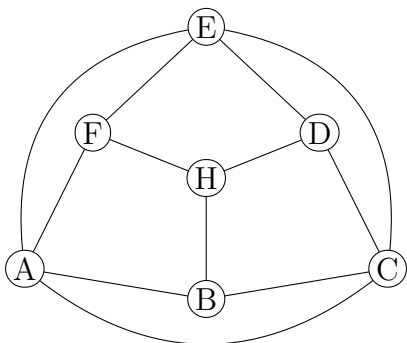
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1. (9 points) What is the chromatic number of graph G (below)? Justify your answer.



2. (6 points) Check the (single) box that best characterizes each item.

$$\sum_{i=1}^{p-1} i \quad \frac{p(p-1)}{2} \quad \frac{(p-1)^2}{2} \quad \frac{p(p+1)}{2} \quad \frac{(p-1)(p+1)}{2}$$

I heated 2 liters of milk in my big pot. 2 liters is _____ how much the pot holds.

an upper bound on ☐ exactly ☐
a lower bound on ☐ not a bound on ☐

Chromatic number of a graph containing a W_n .

≥ 2 ☐ ≤ 3 ☐ $\geq n$ ☐ can't tell ☐

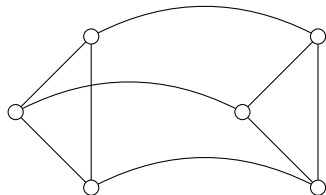
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1. (11 points) Recall that if G is a graph, then $\chi(G)$ is its chromatic number. Let's define the "doubled" version of a graph G as follows: make two copies of G and add an edge joining each pair of corresponding nodes. For example, the doubled version of C_3 looks like:



Suppose that T is the doubled version of a graph G . Describe how $\chi(T)$ is related to $\chi(G)$, justifying your answer. Your answer should handle any choice for G , not just C_3 .

2. (4 points) Check the (single) box that best characterizes each item.

Chromatic number of W_n .2 ☐3 ☐ ≤ 3 ☐ ≤ 4 ☐

10 people can row the canoe but 11 people caused it to sink. 10 is _____ how many people the canoe can carry.

an upper bound on

☐

exactly

☐

a lower bound on

☐

not a bound on

☐