Name:____

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(20 points) The operator $\prod$ is like $\sum$ except that it multiplies its terms rather than adding them. So e.g. $\prod_{p=3}^{5}(p+1) = 4 \cdot 5 \cdot 6$. Also recall that $\sin 2y = 2\sin y \cos y$ for any real number y.

Suppose that x is a real number such that $\sin x$ is non-zero. Use (strong) induction to prove that $\displaystyle\prod_{p=0}^{n-1} \cos(2^p x) = \frac{\sin(2^n x)}{2^n \sin x}$, for any positive integer n.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $f : \mathbb{Z}^+ \to \mathbb{Z}$ is defined by is defined by

$f(1) = 5 \qquad f(2) = -5$

$f(n) = 4f(n-2) - 3f(n-1)$, for all $n \geq 3$

Use (strong) induction to prove that $f(n) = 2 \cdot (-4)^{n-1} + 3$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) A "triangle-free" graph is a graph that doesn't contain any 3-cycles. Use (strong) induction to prove that a triangle-free graph with $2n$ nodes has $\leq n^2$ edges, for any positive integer n. Hint: in the inductive step, remove a pair of nodes joined by an edge. How many edges from those nodes to the rest of the graph?

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Suppose that $f : \mathbb{N} \to \mathbb{Z}$ is defined by

$$f(0) = 1 \qquad f(1) = -5$$

$$f(n) = -7f(n-1) - 10f(n-2), \quad \text{for } n \geq 2$$

Use (strong) induction to prove that $f(n) = (-1)^n \cdot 5^n$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

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(20 points) Use (strong) induction to prove that, for all positive integers n, $x^2 + y^2 = z^n$ has a positive integer solution. (That is, a solution in which x, y, and z are all positive integers.) Hints: (1) notice that $3^2 + 4^2 = 5^2$ and (2) use the solution for $n = k - 2$ (not $n = k - 1$) to build a solution for $n = k$.

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step:

Name:____

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Discussion: **Thursday** **Friday** **10** **11** **12** **1** **2** **3** **4** **5** **6**

(20 points) Suppose that $g : \mathbb{N} \to \mathbb{R}$ is defined by

$$g(0) = 0 \qquad g(1) = \frac{4}{3}$$

$$g(n) = \frac{4}{3}g(n-1) - \frac{1}{3}g(n-2), \quad \text{for } n \geq 2$$

Use (strong) induction to prove that $g(n) = 2 - \dfrac{2}{3^n}$

Proof by induction on n.

Base case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Rest of the inductive step: