Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(10 points) Suppose we have a function f defined (for n a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Express $f(n)$ in terms of $f(n/4^{13})$ (assuming n is large enough that this input hasn't reached the base case). Express your answer using a summation and show your work. Do **not** finish the process of finding the closed form for $f(n)$.

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 4g(n/2) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Express $g(n)$ in terms of $g(n/2^3)$ (where $n \geq 8$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The Fibonacci numbers can be defined recursively by $F(0) = 0$, $F(1) = 1$, and $F(n+1) = F(n) + F(n-1)$ for all integers ...

$n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☐

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Suppose we have a function f defined by

$$\begin{aligned} f(1) &= 5 \\ f(n) &= 3f(n-1) + n^2 \text{ for } n \geq 2 \end{aligned}$$

Express $f(n)$ in terms of $f(n-3)$ (where $n \geq 4$). Show your work and simplify your answer. You do **not** need to find a closed form for $f(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The Fibonacci numbers can be defined recursively by $F(0) = 0$, $F(1) = 1$, and $F(n+2) = F(n) + F(n+1)$ for all integers ...

$n \geq 0$ ☐ $n \geq 1$ ☐ $n \geq 2$ ☐

Name:_____

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(10 points) Suppose we have a function g defined (for n a power of 2) by

$$\begin{aligned} g(1) &= 3 \\ g(n) &= 4g(n/2) + n \text{ for } n \geq 2 \end{aligned}$$

Your partner has already figured out that

$$g(n) = 4^k g(n/2^k) + \sum_{p=0}^{k-1} n2^p$$

Finish finding the closed form for $g(n)$ assuming that n is a power of 2. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(10 points) Suppose we have a function f defined (for n a power of 4) by

$$\begin{aligned} f(1) &= 0 \\ f(n) &= 2f(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Your partner has already figured out that

$$f(n) = 2^k f(n/4^k) + n \sum_{p=0}^{k-1} 1/2^p$$

Finish finding the closed form for $f(n)$ assuming that n is a power of 4. Show your work and simplify your answer. Recall that $\log_b n = (\log_a n)(\log_b a)$.

Name:_____

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Suppose we have a function g defined (for n a power of 4) by

$$\begin{aligned} g(1) &= c \\ g(n) &= 2g(n/4) + n \text{ for } n \geq 4 \end{aligned}$$

Express $g(n)$ in terms of $g(n/4^3)$ (where $n \geq 64$). Show your work and simplify your answer. You do **not** need to find a closed form for $g(n)$.

2. (2 points) Check the (single) box that best characterizes each item.

The chromatic number of the 4-dimensional hypercube Q_4 2 ☐ 3 ☐ 4 ☐ 5 ☐