Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(18 points) Sarah needs to saw a m by n by p inch block of wood into one-inch cubes. (m, n, and p are integers.) The saw can slice a block of wood at any integer position parallel to one of its sides. However, a safety feature prevents her from slicing more than one piece of wood at a time. Use (strong) induction to prove that it takes $mnp - 1$ cuts to divide the block of wood into one-inch cubes, for any sequence of cuts.

The induction variable is named ____ and it is the _____ of/in the block.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(18 points) Recall that a node in a full binary tree is either a leaf or has exactly two children. A Monkey tree is a full binary tree such the two child subtrees of each internal node have heights that differ by at most one. Prove that every Monkey tree of height h contains at least F_{h+1} leaves, where F_k is the kth Fibonacci number. (Recall: $F_0 = 0$, $F_1 = F_2 = 1$)

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(18 points) Here is a grammar G, with start symbols N and P, and terminal symbols a and b.

$$\begin{aligned} N &\rightarrow P\ P \mid a\ b \\ P &\rightarrow N\ P \mid b \end{aligned}$$

Use (strong) induction to prove that any tree matching (aka generated by) grammar G has an even number of leaves if and only if its root has label N.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(18 points) A Camel tree is a binary tree whose nodes contain integers such that

- Every leaf node contains 7, 9, or 12.
- A node with one child contains the same number as its child.
- A node with two children contains the value $xy - y$, where x and y are the values in its children.

Use (strong) induction to prove that the value in the root of a Camel tree is always ≥ 7

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(18 points) UIUC is considering hosting massive hackathons in rooms like the first floor of the Armory. Facilities will need to divide this $100h$ square foot room into h workspaces, each 100 square feet, using expanding partitions. Each end of each partition must be attached to a wall of the room or to another partition. A partition can expand to any length but cannot cross another partition. The partitions are low enough that doors are not required. Use (strong) induction to prove that they will need $h - 1$ partitions, no matter how they arrange the partitions.

The induction variable is named _____ and it is the _____ of/in the room.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step:

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(18 points) Octopus trees are binary trees whose nodes are labelled with strings, such that

- Each leaf node has label `left`, `right`, or `back`
- If a node has one child, it has label α`x` where where α is the child's label. E.g. if the child has label `left` then the parent has `leftx`.
- If a node has two children, it contains $\alpha\beta$ where α and β are the child labels. E.g. if the children have labels `right` and `back`, then the parent has label `rightback`.

Let S(n) be the length of the label on node n. Let L(n) be the number of leaves in the subtree rooted at n. Use (strong) induction to prove that $S(n) \geq 4L(n)$ if n is the root node of any Octopus tree.

The induction variable is named _____ and it is the _____ of/in the tree.

Base Case(s):

Inductive Hypothesis [Be specific, don't just refer to "the claim"]:

Inductive Step: