Name:_____

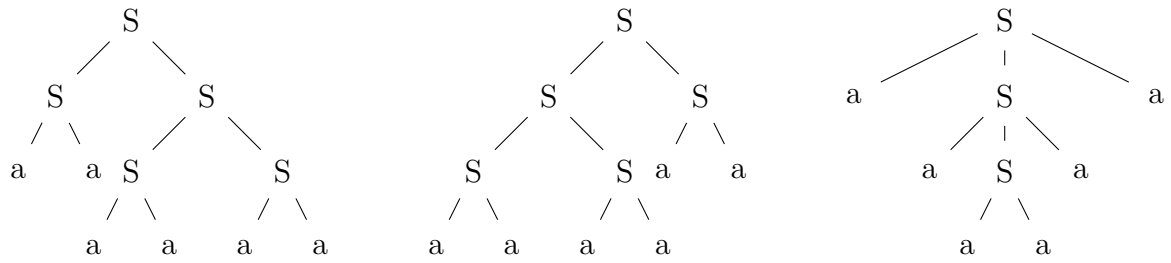
NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbol a. Draw three parse trees for the string $a\ a\ a\ a\ a\ a$ that match this grammar.

$$S \ \rightarrow \ \ S\ S \ \mid \ a\ S\ a \ \mid \ a\ a$$

Solution:



2. (4 points) Check the (single) box that best characterizes each item.

A binary tree of height h has at most $2^{h+1} - 1$ nodes. true [√] false [ ]

The root node of a tree is a leaf. always [ ] sometimes [√] never [ ]

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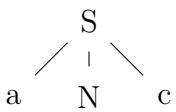
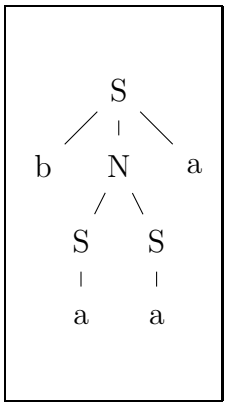
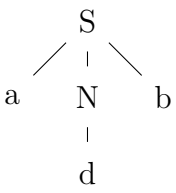
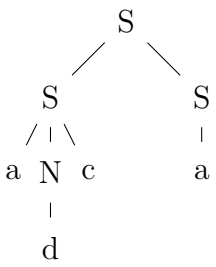
1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, c, and d. Circle the trees that match the grammar.

$$S \rightarrow b\ N\ a \mid a\ N\ c \mid a$$
$$N \rightarrow S\ S \mid d$$









2. (4 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n} 2^k$ $2^{n+1}-1$ ☐ $2^{n+1}-2$ ☑ $2^{n+1}-3$ ☐ 2^n-1 ☐

2^h is ____ the number of leaves in a binary tree of height h.

an upper bound on ☑ exactly ☐

a lower bound on ☐ not a bound on ☐

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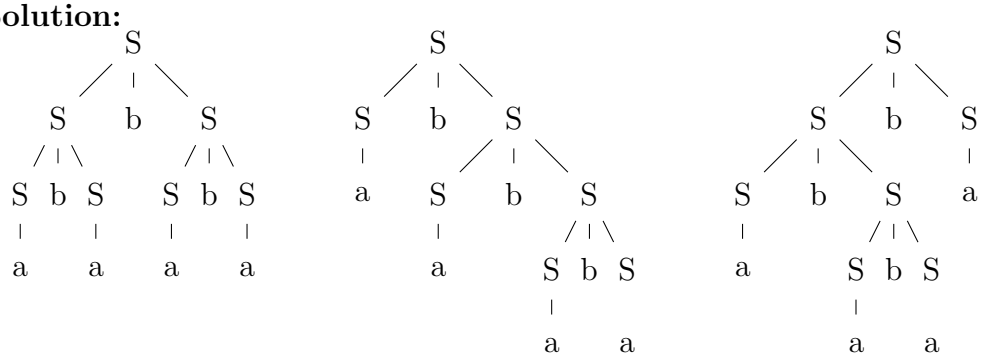
1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string $a\ b\ a\ b\ a\ b\ a$ that match this grammar.

$$S \ \rightarrow \ S\ b\ S \ \mid \ a$$

Solution:



2. (4 points) Check the (single) box that best characterizes each item.

An m-ary tree with i internal nodes has $mi + 1$ nodes total. always ☐ sometimes ☑ never ☐

Total number of leaves in a 3-ary tree of height h 3^h ☐ $\leq 3^h$ ☑ $\frac{1}{2}(3^{h+1} - 1)$ ☐ $3^{h+1} - 1$ ☐

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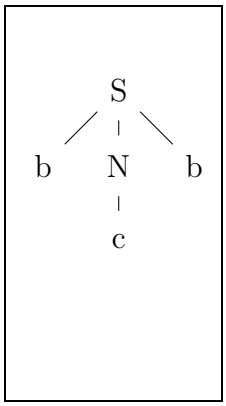
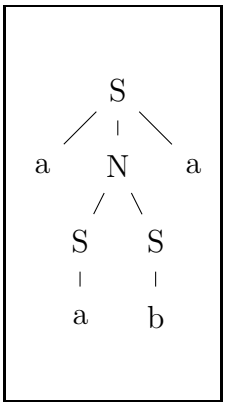
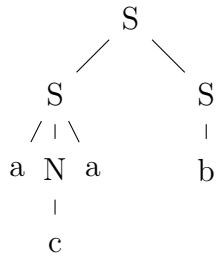
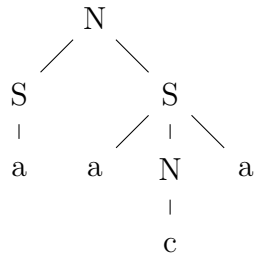
1. (8 points) Here is a grammar with start symbol S and terminal symbols a, b, and c. Circle the trees that match the grammar.

$$S \rightarrow a\ N\ a \mid b\ N\ b \mid a \mid b$$
$$N \rightarrow S\ S \mid c$$









2. (4 points) Check the (single) box that best characterizes each item.

The number of paths between two distinct nodes in an n-node tree. Paths in opposite directions count as the same.	n ☐	$2n$ ☐	$\frac{n(n-1)}{2}$ ☑
	$n(n-1)$ ☐	n^2 ☐	$\frac{n(n+1)}{2}$ ☐

$\sum_{k=0}^{n} 2^k$	$2^n - 2$ ☐	$2^n - 1$ ☐	$2^{n-1} - 1$ ☐	$2^{n+1} - 1$ ☑

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1. (8 points) Consider the following grammar G

   $S \to S\ b\ S \mid a \mid c\ d$

   S is the only start symbol. The terminal symbols are a, b, c, and d.

   Here are two sequences of leaf labels. For each sequence, either draw a tree from grammar G whose leaves have this sequence of labels, or else explain briefly why G cannot generate this sequence of leaf labels.

   $a\ b\ a\ c\ a$

   Solution: Impossible. In this grammar, a c must always be followed by a d, and this one is followed by an a.

   $b\ b\ b\ b\ b$

   Solution: Impossible. Since the only terminal in the string is b, the only rule we could be using is $S \to SbS$. But each time we use this rule, the count of S nodes without children increases by one. This is a problem, since S nodes can't be leaves.

2. (4 points) Check the (single) box that best characterizes each item.

   The level of a leaf node in a full and complete binary tree of height h.

   0 ☐ 1 ☐ $h-1$ ☐ $\leq h$ ☐ h ☑

   Height of a binary tree with 2^n nodes.

   $\leq n-1$ ☐ $\leq n$ ☐ $\leq 2^n$ ☐ $\leq 2^n - 1$ ☑

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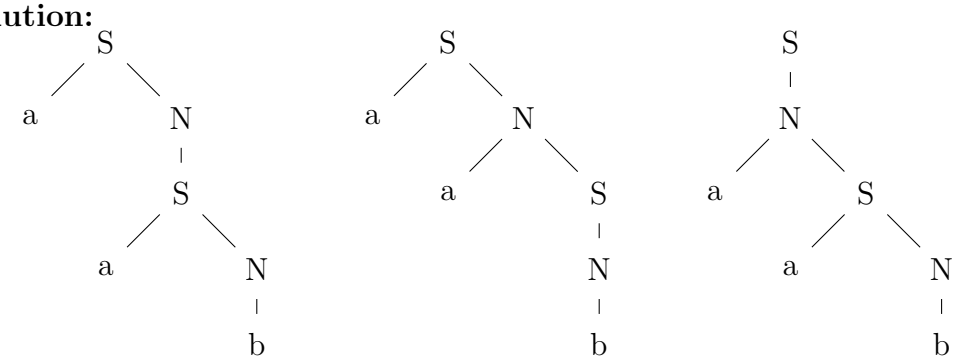
Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (8 points) Here is a grammar with start symbol S and terminal symbols a and b. Draw three parse trees for the string $a\ a\ b$ that match this grammar.

$$\begin{aligned} S &\rightarrow a\ N \mid N \\ N &\rightarrow a\ S \mid S \mid b \end{aligned}$$

Solution:



2. (4 points) Check the (single) box that best characterizes each item.

The number of paths between two nodes in an n-node tree. Paths in opposite directions count as different.

n	☐	$2n$	☐	$\frac{n(n-1)}{2}$	☐
$n(n-1)$	☐	n^2	☑	$\frac{n(n+1)}{2}$	☐

A tree node is an ancestor of itself.

always ☑ sometimes ☐ never ☐