Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a multiple of 3.

   $T(3) = 7$ $T(n) = 2T(n-3) + c$

   (a) The height: $\frac{n}{3} - 1$

   (b) The number of leaves (please simplify): $2^{\frac{n}{3}-1}$

   (c) Total work (sum of the nodes) at level k (please simplify): There are 2^k nodes at level k, each containing value c. So the total work is $c2^k$.

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   3^n $4^{\log_2 n}$ 2^{3n} $3^{\log_2 4}$ $0.1n$ $(5n)!$ $\sqrt{n}$

   Solution:

   $3^{\log_2 4} \ll \sqrt{n} \ll 0.1n \ll 4^{\log_2 n} \ll 3^n \ll 2^{3n} \ll (5n)!$

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1. (7 points) Prof. Flitwick claims that for any functions f and g from the reals to the reals whose output values are always > 1, if $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$ Is this true? Briefly justify your answer.

   Solution: This is not true. Consider $f(x) = x$ and $g(x) = x^2$. Then $\log(g(x)) = 2\log(f(x))$ So it can't be the case that $\log(f(x)) \ll \log(g(x))$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$ $T(n) = 4T(n/2) + n$	$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
	$\Theta(n^2)$	☑	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐
$T(1) = d$ $T(n) = T(n/2) + c$	$\Theta(\log n)$	☑	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
	$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

Suppose $f(n) \ll g(n)$. Is $g(n) \ll f(n)$?	no	☑	perhaps	☐	yes	☐
Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $g(n)$ be $O(f(n))$?	no	☑	perhaps	☐	yes	☐

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $\Theta(h(x))$ and $g(x)$ is $\Theta(h(x))$. Must $f(x) - g(x)$ be $\Theta(h(x))$?

   Solution: This is false.

   Suppose that $g(x) = h(x) = x^2$ and $f(x) = x^2 + x$. Then $f(x) - g(x) = x$, which is not $\Theta(x^2)$.

2. (8 points) Check the (single) box that best characterizes each item.

   $T(1) = d$
   $T(n) = 3T(n-1) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☑

   $T(1) = d$
   $T(n) = 2T(n/2) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☑	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

   Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ ☑		$k = m$ ☐	
$k > m$ ☐		$km = 1$ ☐	

   $n^{log_3 2}$ grows

faster than n ☐		slower than n ☑	
at the same rate as n ☐			

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 3.

   $T(9) = 7$ $T(n) = T\left(\frac{n}{3}\right) + n^2$

   (a) The height: $\log_3 n - 3$

   (b) Number of nodes at level k: One. (This tree does not branch.)

   (c) Value in each node at level k: At level k, the problem size is $\frac{n}{3^k}$. So the value in each node is $(\frac{n}{3^k})^2$.

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $(\sqrt{n})^4$ $200 \log_5 n$ $\log(2^n)$ $2^n + n!$ 7^n 3^{57} $55n \log n$

   Solution:

   $3^{57} \ll 200 \log_5 n \ll \log(2^n) \ll 55n \log n \ll (\sqrt{n})^4 \ll 7^n \ll 2^n + n!$

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(8) = 7$ $T(n) = 4T\left(\frac{n}{2}\right) + n$

   (a) The height: $\log_2 n - 3$

   (b) Total work (sum of the nodes) at level k (please simplify): There are 4^k nodes at level k. Each one contains the value $\frac{n}{2^k}$. So the total for the level is $2^k n$.

   (c) The number of leaves (please simplify): $4^{\log_2 n - 3} = \frac{1}{4^3} 4^{\log_2 n}$
   $4^{\log_2 n} = 4^{\log_4 n \log_2 4} = (4^{\log_4 n})^{\log_2 4} == n^{\log_2 4} = n^2$
   So the number of leaves is $\frac{1}{4^3} n^2$.

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $15n!$ $\log(n^5)$ $127(2^n)$ $n \log_2 4$ 7^n $47n^3$ $20n$

   Solution:

   $\log(n^5) \ll n \log_2 4 \ll 20n \ll 47n^3 \ll 127(2^n) \ll 7^n \ll 15n!$

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for g to be $O(f)$.

   Solution: There are positive reals c and k such that $0 \leq g(x) \leq cf(x)$ for every $x \geq k$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $T(n) = T(n-1) + c$	$\Theta(\log n)$	☐	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☑	$\Theta(n \log n)$	☐
	$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐
$T(1) = d$ $T(n) = T(n/3) + c$	$\Theta(\log n)$	☑	$\Theta(\sqrt{n})$	☐	$\Theta(n)$	☐	$\Theta(n \log n)$	☐
	$\Theta(n^2)$	☐	$\Theta(n^3)$	☐	$\Theta(2^n)$	☐	$\Theta(3^n)$	☐

Suppose $f(n) \ll g(n)$. Is $g(n) \ll f(n)$?	no	☑	perhaps	☐	yes ☐
Suppose $f(n)$ is $\Theta(g(n))$. Will $g(n)$ be $O(f(n))$?	no	☐	perhaps	☐	yes ☑