Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a multiple of 3.

   $T(3) = 7$ $T(n) = 2T(n-3) + c$

   (a) The height:

   (b) The number of leaves (please simplify):

   (c) Total work (sum of the nodes) at level k (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   3^n $4^{\log_2 n}$ 2^{3n} $3^{\log_2 4}$ $0.1n$ $(5n)!$ $\sqrt{n}$

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1. (7 points) Prof. Flitwick claims that for any functions f and g from the reals to the reals whose output values are always > 1, if $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$ Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = c$ $T(n) = 4T(n/2) + n$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n/2) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

Suppose $f(n) \ll g(n)$. Is $g(n) \ll f(n)$?	no ☐	perhaps ☐	yes ☐
Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. Will $g(n)$ be $O(f(n))$?	no ☐	perhaps ☐	yes ☐

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that $f(x)$ is $\Theta(h(x))$ and $g(x)$ is $\Theta(h(x))$. Must $f(x) - g(x)$ be $\Theta(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$
$T(n) = 3T(n-1) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

$T(1) = d$
$T(n) = 2T(n/2) + c$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ ☐ $k = m$ ☐

$k > m$ ☐ $km = 1$ ☐

$n^{log_3 2}$ grows

faster than n ☐ slower than n ☐

at the same rate as n ☐

Name:_____

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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 3.

   $T(9) = 7$ $T(n) = T\left(\frac{n}{3}\right) + n^2$

   (a) The height:

   (b) Number of nodes at level k:

   (c) Value in each node at level k:

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $(\sqrt{n})^4$ $200 \log_5 n$ $\log(2^n)$ $2^n + n!$ 7^n 3^{57} $55n \log n$

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NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

   $T(8) = 7$ $T(n) = 4T\left(\frac{n}{2}\right) + n$

   (a) The height:

   (b) Total work (sum of the nodes) at level k (please simplify):

   (c) The number of leaves (please simplify):

   Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

   $15n!$ $\log(n^5)$ $127(2^n)$ $n \log_2 4$ 7^n $47n^3$ $20n$

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for g to be $O(f)$.

2. (8 points) Check the (single) box that best characterizes each item.

$T(1) = d$ $T(n) = T(n-1) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐
$T(1) = d$ $T(n) = T(n/3) + c$	$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
	$\Theta(n^2)$ ☐	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐

Suppose $f(n) \ll g(n)$.
Is $g(n) \ll f(n)$? no ☐ perhaps ☐ yes ☐

Suppose $f(n)$ is $\Theta(g(n))$.
Will $g(n)$ be $O(f(n))$? no ☐ perhaps ☐ yes ☐