Name:_____

NetID:_____ Lecture: A B

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a multiple of 3.

$$T(3) = 7$$
 $T(n) = 2T(n-3) + c$

- (a) The height:
- (b) The number of leaves (please simplify):
- (c) Total work (sum of the nodes) at level k (please simplify):

Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

 3^n $4^{\log_2 n}$ 2^{3n} $3^{\log_2 4}$ 0.1n (5n)! \sqrt{n}

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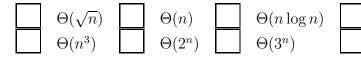
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1. (7 points) Prof. Flitwick claims that for any functions f and g from the reals to the reals whose output values are always > 1, if $f(x) \ll g(x)$ then $\log(f(x)) \ll \log(g(x))$ Is this true? Briefly justify your answer.

2. (8 points) Check the (single) box that best characterizes each item.

T(1) = c	$\Theta(\log n)$	
T(n) = 4T(n/2) + n	$\Theta(n^2)$,



T(1) = d	$\Theta(\log n)$	$\Theta(\sqrt{n})$	$\Theta(n)$	$\Theta(n \log n)$	
T(n) = T(n/2) + c	$\Theta(n^2)$	$\Theta(n^3)$	$\Theta(2^n)$	$\Theta(3^n)$	

Suppose $f(n) \ll g(n)$.				
Is $g(n) \ll f(n)$?	no	perhaps	yes	

Suppose f and g produce only positive outputs and $f(n) \ll g(n)$. no perhaps yes Will g(n) be O(f(n))?

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1. (7 points) Suppose that f, g, and h are functions from the reals to the reals, such that f(x) is $\Theta(h(x))$ and g(x) is $\Theta(h(x))$. Must f(x) - g(x) be $\Theta(h(x))$?

2. (8 points) Check the (single) box that best characterizes each item.

$$T(1) = d$$

$$T(n) = 3T(n-1) + c$$

$$\Theta(\log n)$$

 $\Theta(n^2)$

$\Theta(\sqrt{n})$
$\Theta(n^3)$



$\Theta(n \log n)$
$\Theta(3^n)$

 \mathbf{A}

 \mathbf{B}

$$T(1) = d$$

$$T(n) = 2T(n/2) + c$$

$$\Theta(\log n)$$
 $\Theta(n^2)$

$\Theta(\sqrt{n})$
$\Theta(n^3)$

$$\Theta(n)$$

$$\Theta(2^n)$$

$\Theta(n \log n)$
$\Theta(3^n)$

Dividing a problem of size n into k subproblems, each of size n/m, has the best big- Θ running time when

$$k = m$$

$$km = 1$$

 n^{log_32} grows

faster than n at the same rate as n

slower than n

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Lecture: A B

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10

11 12

1

2 3

5

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6

1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 3.

$$T(9) = 7$$

$$T(n) = T\left(\frac{n}{3}\right) + n^2$$

- (a) The height:
- (b) Number of nodes at level k:
- (c) Value in each node at level k:

Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

 $(\sqrt{n})^4$

 $200\log_5 n$

 $\log(2^n)$

 $2^{n} + n!$

 7^n

 3^{57}

 $55n \log n$

 \mathbf{A}

 \mathbf{B}

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1. (9 points) Fill in key facts about the recursion tree for T, assuming that n is a power of 2.

 $T(8) = 7 T(n) = 4T\left(\frac{n}{2}\right) + n$

- (a) The height:
- (b) Total work (sum of the nodes) at level k (please simplify):
- (c) The number of leaves (please simplify):

Change of base formula: $\log_b n = (\log_a n)(\log_b a)$

2. (6 points) Write the following functions in the boxes so that f is to the left of g if and only if $f(n) \ll g(n)$.

15n! $\log(n^5)$ 127(2ⁿ) $n\log_2 4$ 7ⁿ 47n³ 20n

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1. (7 points) Suppose that f and g are functions from the reals to the reals. Define precisely what it means for g to be O(f).

2. (8 points) Check the (single) box that best characterizes each item.

$$T(1) = d$$

$$T(n) = T(n-1) + c$$

$$\Theta(\log n)$$

$$\Theta(n^2)$$

$$\Theta(n)$$

$$\Theta(2^n)$$

$$\Theta(n \log n)$$

$$\Theta(3^n)$$

 \mathbf{B}

$$T(1) = d$$

$$T(n) = T(n/3) + c$$

$$\Theta(\log n)$$
 $\Theta(n^2)$

$\Theta(\sqrt{n})$
$\Theta(n^3)$

$$\Theta(n)$$

$$\Theta(2^n)$$

$\Theta(n \log n)$
$\Theta(3^n)$

Suppose $f(n) \ll g(n)$. Is $g(n) \ll f(n)$?

no	

perhaps yes

Suppose f(n) is $\Theta(g(n))$. Will g(n) be O(f(n))?

	_	
no		
110		

perhaps

yes