Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(8 points) The Google interviewer suggests that $\binom{n}{k}$ can be computed very efficiently using the equation $\binom{n}{k} = \frac{n+1-k}{k}\binom{n}{k-1}$. Is this formula correct? Assume $k > 0$. Briefly justify your answer.

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every dog d, if d is a terrier, then d is not large and d is noisy.

(2 points) Check the (single) box that best characterizes each item.

V is the vertex set of a tree with n edges. $|\mathbb{P}(V)| =$

☐ 2^{n-1} ☐ 2^n ☐ not determined ☐

☐ 2^{n+1} ☐ n

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) Every hacker is a black hat or a white hat (and not both). White hats always tell the truth. Black hats always lie. Alfred says to you "I am a black hat." Use proof by contradiction to show that Alfred is not a hacker.

(6 points) If $x, y, z \in \mathbb{N}$, how many solutions are there to the equation $x + y + z = 25$?

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(8 points) A triomino is a triangular tile with a number on each edge. In our set of triominos, the numbers range from 0 to 5. So possible tiles include 5-3-4, 0-4-4, and 3-3-3. Tiles can be turned over: Also notice that a tile is the same if you rotate it. So 5-3-4 is the same tile as 3-4-5, 4-5-3, and also 5-4-3. How many distinct tiles are in our set?

(5 points) State the negation of the following claim, moving all negations (e.g. "not") so that they are on individual predicates.

For every tiger k, if k is orange, then k is large and k is not friendly.

(2 points) Check the (single) box that best characterizes each item.

The number of ways to select a set of 4 flowers chosen from 17 possible varieties (zero or more of each variety).	$\binom{17}{5}$ ☐	$\binom{20}{4}$ ☐	$\binom{20}{3}$ ☐
	$\binom{17}{4}$ ☐	$\binom{21}{4}$ ☐	$\frac{17!}{4!}$ ☐

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) Use proof by contradiction to show that, in any group of 7 people, there is at least one person who knows an even number of people. (Assume that "knowing someone" is symmetric.)

(6 points) Margaret's home is defended from zombies by wallnuts, peashooters, and starfruit. She has a row of 20 pedestals on which they can stand, and she needs to use at least one starfruit. How many options does she have for the placing defenders on the pedestals?

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) Use proof by contradiction to show that $\sqrt{\sqrt{2}}$ is not rational. (You may use the fact that $\sqrt{2}$ is not rational.)

(6 points) In the town of West Fork, the streets are laid out in a uniform square grid. Alvin's school lies 6 blocks east and 9 blocks north of his house. So (since there are no diagonal roads) he travels 15 blocks to school. How many different 15-block paths can he choose from? Show your work or justify your answer.

Name:_____

NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(9 points) Use proof by contradiction to show that there are no positive integer solutions to the equation $x^2 - y^2 = 10$.

(6 points) Margaret's home is defended from zombies by wallnuts, peashooters, and starfruit. She has room to place 20 of these on her lawn. How many options does she have for her set of defenders?