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Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(7 points) Can we create a set C such that C is a partition of $\mathbb{R}$ but $|C|$ is finite? Give a specific set C that works or briefly explain why it's impossible.

Solution: Yes. Suppose that C contains exactly two sets: the negative reals and the non-negative reals. Then $|C| = 2$, which is finite.

(8 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cap \mathbb{P}(B) = \mathbb{P}(A \cap B)$ always [√] sometimes [ ] never [ ]

If $n \geq k \geq 0$, then $\binom{n}{k} = \binom{n}{n-k}$ true [√] true for some n and k [ ] false [ ]

$\binom{n}{0}$ -1 [ ] 0 [ ] 1 [√] 2 [ ] n [ ] undefined [ ]

$\mathbb{P}(A) \cap \mathbb{P}(B) = \emptyset$ always [ ] sometimes [ ] never [√]

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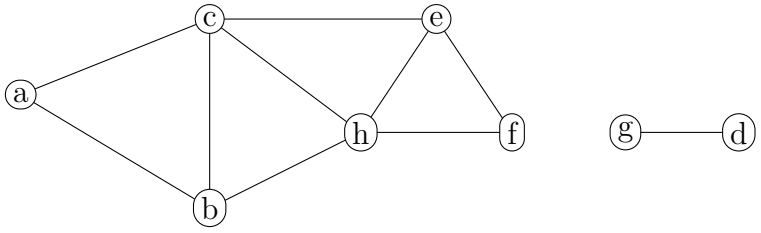
Graph G is shown below with set of nodes V and set of edges E.



Let $f : V \to \mathbb{P}(V)$ such that $f(n) = \{v \in V \mid$ there is a cycle containing n and $v\}$.
Let $T = \{f(n) \mid n \in V\}$.

(6 points) Fill in the following values:

$|E| =$

Solution: 10

$f(b) =$

Solution: $\{a, b, c, e, f, h\}$

$f(h) =$

Solution: $\{a, b, c, e, f, h\}$

(7 points) Is T a partition of V? For each of the three conditions required to be a partition, explain why T does or doesn't satisfy that condition.

Solution: No, it is not a partition of V. There is no partial overlap (good). However, T contains the empty set because $f(g) = \emptyset$. And some vertices (e.g. k) do not belong to any cycles and therefore aren't in any elements of T.

(2 points) Check the (single) box that best characterizes each item.

$\mathbb{P}(A) \cup \mathbb{P}(B) = \mathbb{P}(A \cup B)$ always ☐ sometimes ☑ never ☐

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Let $f : \mathbb{R}^2 \to \mathbb{P}(\mathbb{R}^2)$ be defined by $f(x, y) = \{(p, q) \in \mathbb{R}^2 \mid x^2 + y^2 = p^2 + q^2\}$
Let $T = \{f(x, y) \mid (x, y) \in \mathbb{R}^2\}$.

(6 points) Answer the following questions:

$f(0, 0) =$

Solution: $\{(0, 0)\}$

Describe (at a high level) the elements of $f(0, 36)$:

Solution: The circle centered on the origin with radius 36.

The cardinality of (aka the number of elements in) T is:

Solution: infinite

(7 points) Is T a partition of $\mathbb{R}^2$? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: Yes. The output of f is never the empty set. None of these circles (plus the dot at the origin) overlaps any of the others. And jointly they cover all of the plane.

(2 points) Check the (single) box that best characterizes each item.

Let A be a non-empty set,
$\{A\}$ is a partition of A. always [√] sometimes [ ] never [ ]

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(7 points) Suppose that A and B are disjoint sets, C_A is a partition of A and C_B is a partition of B. Is $C_A \cup C_B$ a partition of $A \cup B$? Briefly justify your answer.

Solution: Yes. Each element of A belongs to exactly one element of C_A. Each element of B belongs to exactly one element of C_B. Since A and B are disjoint, there can't be any partial overlap between the elements of C_A and C_B. Since neither C_A nor C_B contains the empty set, their union can't contain it either.

(8 points) Check the (single) box that best characterizes each item.

$|\mathbb{P}(\mathbb{P}(\emptyset))|$ 0 ☐ 1 ☐ 2 ☑ 3 ☐ 4 ☐ undefined ☐

If $f : \mathbb{P}(\mathbb{Q}) \to \mathbb{N}$ then $f(3)$ is a rational ☐ a power set of rationals ☐ a set of rationals ☐ undefined ☑

$|\{\emptyset\}|$ 0 ☐ 1 ☑ 2 ☐ 3 ☐ 4 ☐ undefined ☐

$\mathbb{P}(A \cup B) = \mathbb{P}(A) \cup \mathbb{P}(B)$ always ☐ sometimes ☑ never ☐

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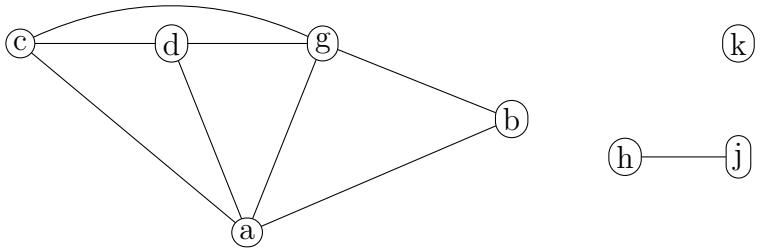
Graph G is at right.
V is the set of nodes.
E is the set of edges.
ab (or ba) is the edge between a and b.



Let $f : V \to \mathbb{P}(V)$ be defined by $f(n) = \{v \in V \mid \text{there is a path from n to v}\}$. And let $T = \{f(n) \mid n \in V\}$.

6 points) Fill in the following values:

$f(k) =$ **Solution:** $\{k\}$

$f(d) =$ **Solution:** $\{a, b, c, d, g\}$

$T =$ **Solution:** $\{\{a, b, c, d, g\}\{h, j\}\{k\}\}$

(7 points) Is T a partition of V? For each of the conditions required to be a partition, briefly explain why T does or doesn't satisfy that condition.

Solution: Yes, T is a partition of V. T does not contain the empty set. The members of T contain all nodes in V, with no partial overlap

(2 points) Check the (single) box that best characterizes each item.

$\binom{n}{1}$ -1 ☐ 0 ☐ 1 ☐ 2 ☐ n ☑ undefined ☐

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(7 points) Can a set A be a partition of the empty set? Briefly justify your answer.

Solution: Yes. Suppose that A is the empty set. Then each member of the empty set is in exactly one member of A. Also, the empty set is not an element of A.

(8 points) Check the (single) box that best characterizes each item.

Pascal's identity states that $\binom{n+1}{k}$ is equal to $\binom{n}{k} + \binom{n}{k+1}$ ☐ $\binom{n}{k} + \binom{n-1}{k}$ ☐ $\binom{n}{k} + \binom{n}{k-1}$ ☑

$\mathbb{P}(A \cap B) \subseteq \mathbb{P}(A \cup B)$ always ☑ sometimes ☐ never ☐

If $f : \mathbb{R} \to \mathbb{P}(\mathbb{Z})$ then $f(17)$ is an integer ☐ a set of integers ☑ one or more integers ☐ a power set ☐

A partition of a set A contains A always ☐ sometimes ☑ never ☐