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(15 points) Recall that a phone lattice is a state diagram representing sequences of letters. Each edge in a phone lattice has a single letter on it. In a "deterministic" state diagram, if you look at any state s and any letter a, there is never more than one edge labelled a leaving state s.

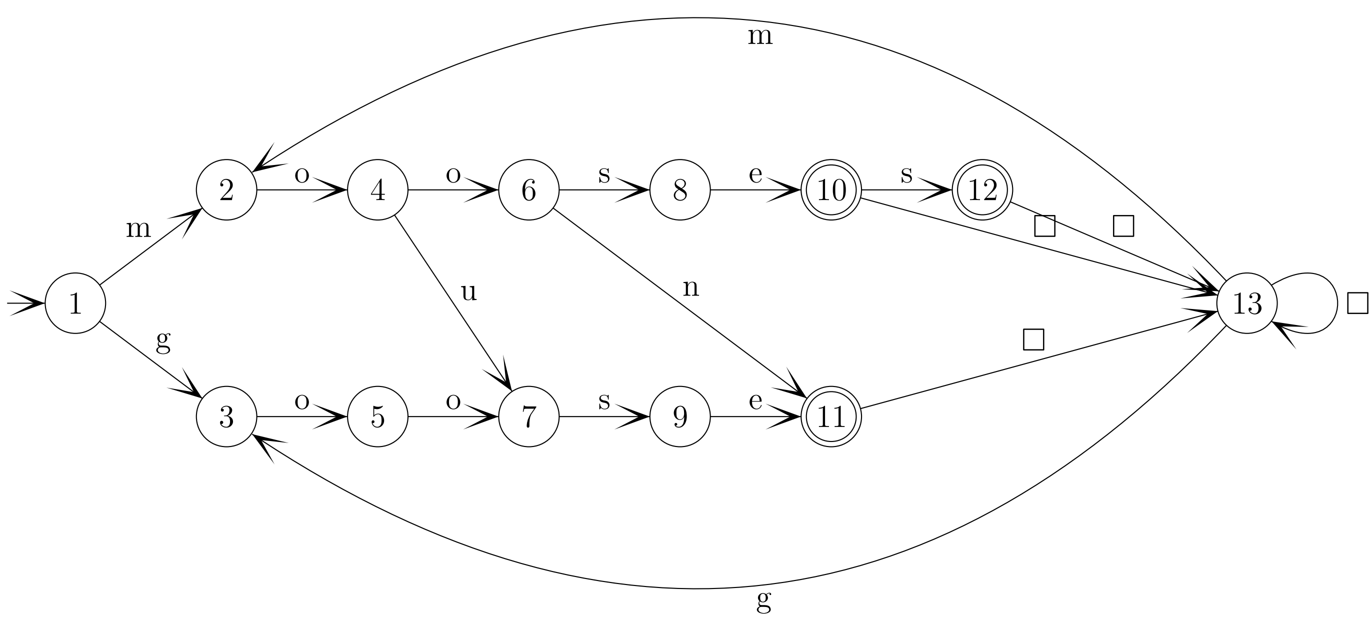
Draw a deterministic phone lattice that accepts sequences of words from the following list, separated by spaces.

mouse, moose, mooses, goose, moon

A sample input might be `"moon mooses     goose"`.

Your lattice should allow one or more spaces between each pair of consecutive words. Show the space character as □ in your lattice. Do not allow any spaces at the start or end of the sequence. Use no more than 16 states and, if you can, no more than 14.

Solution:



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(5 points) An RGB ring is a 3-cycle, each of whose nodes contains a color label (red, green, or blue) plus a rational number in the range $[0, 1]$. Is the set of all RGB rings countable or uncountable? Briefly justify your answer.

Solution: This set is countable. An RGB ring can be represented as a triple of rational numbers plus a triple of color labels. The rational numbers are countable, as is the set of three color labels. So this is a finite product of countable sets, thus countable.

(10 points) Check the (single) box that best characterizes each item.

Every function from the integers to the integers has a corresponding finite-length formula. true ☐ false ☑ not known ☐

$\mathbb{P}(\mathbb{R})$ has the same cardinality as $\mathbb{R}$. true ☐ false ☑ not known ☐

A subset of a countable set is countable. true ☑ false ☐

All circles in the real plane. finite ☐ countably infinite ☐ uncountable ☑

We can build a program that decides whether an input program halts. true ☐ false ☑ not known ☐

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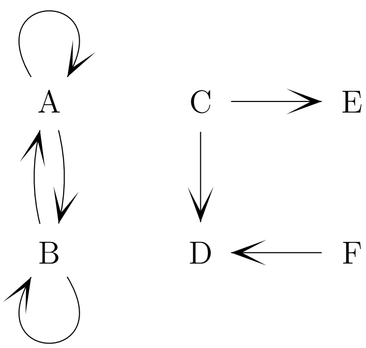
(5 points) Check all boxes that correctly characterize this relation on the set $\{A, B, C, D, E, F\}$.



Reflexive: ☐ Irreflexive: ☐

Symmetric: ☐ Antisymmetric: ☐

Transitive: [√]

(10 points) Check the (single) box that best characterizes each item.

$p \vee q \equiv \neg p \rightarrow q$ true [√] false ☐

$\gcd(p, q) = \frac{pq}{\text{lcm}(p, q)}$ (p and q positive integers) always [√] sometimes ☐ never ☐

$|A - B| = |A| - |B|$ true for all sets A ☐ true for some sets A [√] false for all sets A ☐

A function from $\mathbb{R}$ to $\mathbb{R}$ is strictly increasing if and only if it is one-to-one. true ☐ false [√]

All elements of M are also elements of X. $M = X$ ☐ $M \subseteq X$ [√] $X \subseteq M$ ☐

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(5 points) Is the graph C_7 bipartite? Briefly justify your answer.

Solution: No, it isn't bipartite. As you walk around the cycle, you must assign nodes to the two subsets in an alternating manner. But there's no way to assign the last (odd) node.

(10 points) Check the (single) box that best characterizes each item.

$\sum_{k=-2}^{n} k^2$ — $\sum_{p=0}^{n+2}(p+2)^2$ ☐ $\sum_{p=0}^{n-2}(p-2)^2$ ☐ $\sum_{p=0}^{n+2}(p-2)^2$ ☑ $\sum_{p=0}^{n+2}p^2$ ☐

The Fibonacci numbers can be defined recursively by $F(0) = 0$, $F(1) = 1$, and $F(n+1) = F(n) + F(n-1)$ for all integers ... — $n \geq 0$ ☐ $n \geq 1$ ☑ $n \geq 2$ ☐

$n^{1.5}$ is — $\Theta(n^{1.414})$ ☐ $O(n^{1.414})$ ☐ neither of these ☑

Total number of leaves in a full and complete 5-ary tree of height h — 5^h ☑ $\leq 5^h$ ☐ $\geq 5^h$ ☐ $5^{h+1} - 1$ ☐

$T(1) = c$
$T(n) = 4T(n/2) + n$

$\Theta(\log n)$ ☐	$\Theta(\sqrt{n})$ ☐	$\Theta(n)$ ☐	$\Theta(n \log n)$ ☐
$\Theta(n^2)$ ☑	$\Theta(n^3)$ ☐	$\Theta(2^n)$ ☐	$\Theta(3^n)$ ☐