Name:_____

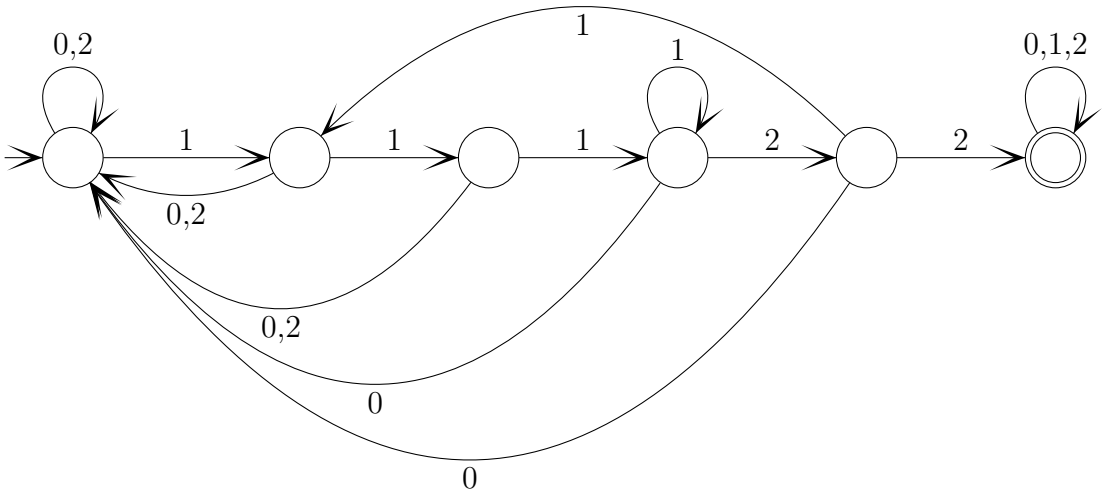
NetID:_____ **Lecture: A B**

Discussion: Thursday Friday 10 11 12 1 2 3 4 5 6

(15 points) Professor Martinez needs a state machine that will recognize the sequence 11122 when typed on a keypad. Specifically, it must read any sequence of the digits 0, 1, and 2. It should move into a final state immediately after seeing 11122, and then remain in that final state as further characters come in. For efficiency, the state machine must be deterministic, i.e. if you look at any state s and any action a, there is **exactly** one edge labelled a leaving state s.

Draw a deterministic state diagram that will meet his needs, using no more than 9 states and, if you can, no more than 6.

Solution:



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(5 points) Let A be the set of all relations on $\mathbb{N}$. Is A countable or uncountable? Briefly justify your answer.

Solution: This set is uncountable. Let A' be the subet of A containing only the relations where a number is never related to a different number. To specify an element of A', we have to specify whether each natural number is related to itself. That is, we create a function the natural numbers to $\{\text{true}, \text{false}\}$. We saw in class that the set of functions from the natural numbers to a 2-element set is uncountable. So A' is uncountable and therefore A is uncountable.

(10 points) Check the (single) box that best characterizes each item.

The rational numbers have the same cardinality as the reals.	true	☐	false	☑	not known	☐	
A product of countable sets is countable.	true	☐	false	☐	true for finite products	☑	
All possibilities for what could be stored on your laptop's hard drive.	finite	☑	countably infinite	☐	uncountable	☐	
The set of all prime numbers.	finite	☐	countably infinite	☑	uncountable	☐	
Every mathematical function has a finite formula.	true	☐	false	☑	not known	☐	

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(5 points) Is the following claim true? Informally explain why it is, or give a concrete counter-example showing that it is not.

Claim: For all positive integers a, b, and c, if $\gcd(a, b) = 1$ and $\gcd(b, c) = 1$, then $\gcd(a, c) = 1$.

Solution: This is false. Consider $a = c = 3$ and $b = 2$. Then a and b have no common factors, i.e. $\gcd(a, b) = 1$. Also b and c have no common factors, i.e. $\gcd(b, c) = 1$. But $\gcd(a, c) = 3$.

(10 points) Check the (single) box that best characterizes each item.

$\forall x \in \mathbb{R}$,
if $x^2 > 100$, then $|x| \geq 10$. true [√] false [ ]

$\{1, 2\} \cap \emptyset =$

$\emptyset$ [√] $\{(1, \emptyset), (2, \emptyset)\}$ [ ] $\{1, 2, \emptyset\}$ [ ]

$\{\emptyset\}$ [ ] $\{1, 2\}$ [ ] undefined [ ]

If xRy is never true, then the relation R is

symmetric [ ] antisymmetric [ ]
both [√] neither [ ]

If a function from $\mathbb{R}$ to $\mathbb{R}$ is increasing, it must be one-to-one. true [ ] false [√]

Chromatic number of a graph with no cycles and at least one edge

1 [ ] 2 [√] 3 [ ] can't tell [ ]

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(5 points) Suppose that $f : \mathbb{N} \to \mathbb{N}$ is such that $f(n) = n^2$. Give a recursive definition of f

Solution:

$f(0) = 0$, and $f(n+1) = f(n) + 2n + 1$ for $n \geq 0$.

You could also have used $f(n) = f(n-1) + 2n - 1$ for $n \geq 1$.

(10 points) Check the (single) box that best characterizes each item.

$\sum_{k=1}^{n} \frac{1}{2^k}$ $1 - (\frac{1}{2})^{n-1}$ ☐ $2 - (\frac{1}{2})^{n}$ ☐ $1 - (\frac{1}{2})^{n}$ ☑ $2 - (\frac{1}{2})^{n-1}$ ☐

Dividing a problem of size n into k subproblems, each of size n/m, has the best big-Θ running time when

$k < m$ ☑ $k = m$ ☐

$k > m$ ☐ $km = 1$ ☐

3^n is $\Theta(5^n)$ ☐ $O(5^n)$ ☑ neither of these ☐

The diameter of a full, complete 7-ary tree of height h.

$\leq h$ ☐ h ☐ $h+1$ ☐

$2h$ ☑ $7h$ ☐ $7h+1$ ☐

$\binom{k}{k-1}$ 1 ☐ 2 ☐ k-1 ☐ k ☑ undefined ☐